

ON SEMIGROUPS OF TRANSFORMATIONS WITH IDEMPOTENT COMPLEMENT WHOSE RESTRICTIONS BELONG TO A GIVEN SEMIGROUP

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ABSTRACT

Let $X_n = \{1, \dots, n\}$ for any $n \in \mathbb{Z}^+$, and let P_n , T_n , I_n and S_n denote the partial transformation semigroup, (full) transformation semigroup, symmetric inverse semigroup and symmetric group on the set X_n , respectively. In this study, we introduce and examine some new semigroups defined as

$$\begin{aligned} IS_{(n,m)}^f &= \{ \alpha \in I_n : \alpha|_{X_m} \in S_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \}, \\ II_{(n,m)}^f &= \{ \alpha \in I_n : \alpha|_{X_m} \in I_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \}, \\ PS_{(n,m)}^f &= \{ \alpha \in P_n : \alpha|_{X_m} \in S_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \}, \\ PT_{(n,m)}^f &= \{ \alpha \in P_n : \alpha|_{X_m} \in T_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \}, \\ PI_{(n,m)}^f &= \{ \alpha \in P_n : \alpha|_{X_m} \in I_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \}, \\ PP_{(n,m)}^f &= \{ \alpha \in P_n : \alpha|_{X_m} \in P_m; \text{dom}(\alpha) \setminus X_m \subseteq \text{fix}(\alpha) \} \end{aligned}$$

for $1 \leq m \leq n-1$ where $\text{dom}(\alpha) = \{x \in X_n : x\alpha = y \text{ for any } y \in X_n\}$ and $\text{fix}(\alpha) = \{x \in \text{dom}(\alpha) : x\alpha = x\}$.

Keywords Idempotent element · Partial (Full) transformation semigroup · Symmetric inverse semigroup · Symmetric group

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