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GPBiCG(m, ℓ, k): An Extension of GPBiCG(m, ℓ) with Preliminary Adaptive Parameter Selection Strategies

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Abstract

We propose a new Krylov subspace method, denoted GPBiCG(m, ℓ, k), for solving large sparse non-Hermitian linear systems. The method combines features of BiCGSTAB [2], GPBiCG [3], and LOPBiCG [4] within a unified framework. Each cycle consists of m iterations of BiCGSTAB, followed by ℓ iterations of GPBiCG and k iterations of LOPBiCG. The proposed algorithm extends the GPBiCG(m, ℓ) method of Fujino [1] by introducing an additional parameter that increases the flexibility of the residual polynomial construction.

Numerical experiments are performed on a collection of benchmark sparse matrices arising from scientific and engineering applications. The influence of the parameters m, ℓ, and k on convergence behavior is investigated, and the performance of GPBiCG(m, ℓ, k) is compared with BiCGSTAB, BiCGSTAB2, GPBiCG, and GPBiCG(m, ℓ). The results show that, for suitably selected parameter combinations, GPBiCG(m, ℓ, k) can achieve faster convergence and improved robustness on several test problems.

Because the effectiveness of the method depends strongly on the parameter choice, we also examine preliminary adaptive parameter-selection strategies based on cumulative residual reduction criteria. Although these strategies can identify effective parameter combinations in some cases, their performance remains sensitive to the initial parameter settings and does not yet consistently reproduce the best fixed configurations obtained through offline parameter selection.

These results indicate that GPBiCG(m, ℓ, k) provides a promising extension of existing product-type Krylov subspace methods. At the same time, the development of reliable and fully automatic parameter-selection procedures remains an important topic for future research.

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***K*eywords** Non-Hermitian linear systems · Krylov subspace methods · GPBiCG · Residual polynomials · Parameter selection strategies

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