QUANTUM CALCULUS AND FACTORIZATION OF FIBONACCI NUMBERS

Oktay K. Pashaev*

Department of Mathematics, Izmir Institute of Technology, 35430, Türkiye

ABSTRACT

Factorization of q-deformed numbers $[n]_q$ and their generalizations in the set of the deformed prime numbers with base parameters as powers of q is established. We start from simple identity

$$(1+q)(1+q^2)(1+q^4)...(1+q^{2^n}) = \frac{1-q^{2^{n+1}}}{1-q},$$

which can be interpreted it as factorization of q-numbers

$$\prod_{k=0}^{n-1} [2]_{q^{2^k}} = [2^n]_q$$

and generalize it for arbitrary number p. For $|q| < 1$ it includes an infinite product factorization of $[\infty]_q$ in q-deformed primes

$$\prod_{k=0}^{\infty} (1 + q^{3^k} + q^{2\cdot 3^k}) = \frac{1}{1-q} = \prod_{k=0}^{\infty} [3]_{q^{3^k}}$$

For quantum calculus with two basis Q and q, factorization formulas include deformed numbers with sequence of powers of Q and q. As a specific case with $Q = \varphi$ and $q = \varphi'$ being the Golden and the Silver ratio, we get factorization formula for Fibonacci number F_N with arbitrary positive integer $N = p_1^{k_1} p_2^{k_2} ... p_n^{k_n}$ to the set of integer numbers

$$F_N = \prod_{m_1=0}^{k_1-1} \prod_{m_2=0}^{k_2-1} ... \prod_{m_n=0}^{k_n-1} F_{p_1}^{(p_1^{m_1})} \cdot F_{p_2}^{(p_1^{k_1} p_2^{m_2})} \cdot ... \cdot F_{p_n}^{(p_1^{k_1} p_2^{k_2} ... p_{n-1}^{k_{n-1}} p_n^{m_n})}$$

This set of integers is represented by higher Fibonacci numbers [1] or Fibonacci divisors $F_{p_l}^{(k)}$ [2] of prime numbers p_l, with index k given by powers of p_l. Due to divisibility of Fibonacci number F_{nk} by F_k, so that $F_{nk} : F_k = F_n^{(k)}$, this factorization gives product of positive integer numbers.

***K*eywords** Quantum calculus · Fibonacci numbers · Fibonacci divisors · Factorization

References

[1] Pashaev O.K. and Nalci S., Golden quantum oscillator and Binet-Fibonacci calculus, J. Phys. A: Math. Theor., 45, 015303, 2012.

[2] Pashaev O.K., Quantum calculus of Fibonacci divisors and infinite hierarchy of bosonic-fermionic Golden quantum oscillators, Int. J. Geometric Methods in Modern Physics, 18: 2150075, 2021.

Corresponding Author's E-mail: oktaypashaev@iyte.edu.tr