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ON THE CONDITIONING OF VANDERMONDE SYSTEMS WITH MOCK-CHEBYSHEV NODES

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ABSTRACT

The polynomial interpolation problem defined on a set of distinct nodes and represented in a monomial basis results in a linear system of equations with a Vandermonde matrix. A difficulty with a Vandermonde matrix is that when constructed with real nodes, it is generally quite ill-conditioned [1], even for moderately low degrees. This ill-conditioning may vary notably depending on the distribution of the points. The general recommendation is to use the highly non-uniform Chebyshev nodes, but the problem remains when experimental data is available only at equally spaced points. In such cases, polynomial interpolation becomes unreliable due to the Runge phenomenon and is also numerically ill-conditioned. To address these challenges, an effective strategy is to select mock-Chebyshev points from a dense set of uniformly spaced nodes, thereby capturing the favorable interpolation properties of Chebyshev nodes [2, 3, 4].

In this study, we investigate the condition number of the generalized Vandermonde matrix and demonstrate that it can be reduced by employing mock-Chebyshev nodes, similarly to the case of Chebyshev–Lobatto nodes.

***K*eywords** Vandermonde matrix · Condition numbers · Mock-Chebyshev nodes

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