ON CHARACTERIZATION OF A TYPE OF SEMI-SYMMETRIC METRIC CONNECTION ON A SPACETIME

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ABSTRACT

General relativity, which has disclosed the basic connection between physics and the geometry of spacetimes, is one of the most successful physics theories of twentieth Century. In addition to its crucial importance in theoretical studies, general relativity has found success in technology when applied to our daily lives.

A time-oriented, connected and four-dimensional Lorentzian manifold was modeled by both general relativity spacetime and cosmology.

The geometry of Lorentzian manifolds is used to investigate the behavior of vectors on the manifold. Lorentzian manifolds are emerging as the most effective study model to explain the general relativity.

A spacetime is a Lorentzian manifold M with the Lorentzian metric g of signature $(-,+,+,+)$ which permits a globally timelike vector field. Different types of spacetimes have been studied in various ways, such as ([1]-[5]).

Let (M,g) be a connected and time-oriented Lorentzian manifold. If $M = -I \times_{\phi^2} M^*$, where I is the open interval of $\Re$, ϕ is a smooth function (or scale factor, or warping function) and M^* is considered as a three-dimensional Riemannian manifold, then M is named as a generalized Robertson-Walker spacetime [6]. If we consider that M^* is a three-dimensional Riemannian manifold with constant curvature, then the generalized Robertson-Walker spacetime reduces to Roberston-Walker spacetime. Some examples of special spacetimes related by generalized Robertson-Walker spacetimes are de Sitter spacetime, Einstein de Sitter spacetime, static spacetime and Friedmann cosmological models.

A perfect fluid spacetime is a four-dimensional spacetime whose non-vanishing Ricci tensor R_{ij} is of the form

$$R_{ij} = \alpha g_{ij} + \beta w_i w_j$$

assuming that α, β are smooth functions, g is the Lorentzian metric and w_i is the velocity vector satisfying the condition $w_i w^i = -1$.

In a perfect fluid spacetime, the energy momentum tensor T_{ij} is given by

$$T_{ij} = (p+\sigma) w_i w_j + p g_{ij}$$

where σ and p denote the energy density and the isotropic pressure, respectively and w_i is a non-vanishing vector.

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The Einstein field equations without cosmological constant are presented by

$$R_{ij} - \frac{R}{2} g_{ij} = kT_{ij}$$

where R denotes the scalar curvature and k indicates the gravitational constant.
Let $\bar{\nabla}$ represent a linear connection on a Lorentzian manifold (M, g) of dimension n. This connection is referred to a semi-symmetric if the torsion tensor

$$\bar{T}^k_{ij} = \delta^k_j w_i - \delta^k_i w_j$$

where w_i is the associated vector of the linear connection $\bar{\nabla}$.
A linear connection $\bar{\nabla}$ is said to be metric if $\bar{\nabla}_k g_{ij} = 0$, otherwise it is non-metric. Hayden [7] studied the properties of the metric connection on a Riemannian manifold. A systematic study of semi-symmetric connection on a Riemannian manifold was given by Yano [8]. Recently, De et al. [9] have studied the properties of Lorentzian manifolds endowed with the semi-symmetric metric connection.

The main purpose of this paper is to investigate the general properties of spacetime with a semi-symmetric metric connection. Firstly, we show that a spacetime equipped with a semi-symmetric metric w-connection is a dark matter if the Ricci tensor with respect to the connection $\bar{\nabla}$ vanishes. Also, we prove that if the curvature tensor of such a spacetime with respect to the linear connection vanishes then this spacetime is conformally flat.

In the other parts of this paper, we prove that such a spacetime under some conditions becomes Robertson-Walker spacetime or Yang-Pure spacetime or de Sitter spacetime. Finally, we apply these spacetimes to general relativity and we discuss their physical interpretations of some geometric results.

***K*eywords** Generalized Robertson-Walker spacetime · perfect fluid · general relativity · semi-symmetric metric connection · quasi-constant curvature · conformal curvature tensor.

References

[1] A.M. Blaga, Solitons and geometrical structures in a perfect fluid spacetime, Rocky Moun.J.Math., 50: 2020, 41-53.

[2] C.A. Mantica, L.G. Molinari and U.C. De, A condition for a perfect fluid space-time to be a generalized Robertson-Walker space-time, 57: 2016, 022508.

[3] U.C. De and Y.J. Suh, Some characterizations of Lorentzian manifolds, Int.J.Geom.Meth.Mod.Phys., 16(1): 2019, 1950016.

[4] B.Y. Chen, A simple characterization of generalized Robertson-Walker spacetimes, Gen.Rel.Grav. 46: 2014, 1833.

[5] B.Y. Chen, Differential Geometry of warped product manifolds and submanifolds, World Sci.Publ. Hackensack, 2017.

[6] L. Alias, A. Romero and M. Sanchez, Uniqueness of complete spacelike hypersurfaces of constant mean curvature in generalized Roberston-Walker spacetimes, Gen.Rel.Grav., 27: 1995, 71-84.

[7] H.A. Hayden, Subspace of space with torsion, Proc. London Math.Soc., 34: 1932, 27-50.

[8] K. Yano, On semi-symmetric metric connections, Rev.Roum.Mth.Pures Appl., 15: 1970, 1579-1586.

[9] U.C. De, K. De and S. Güler, Characterizations of Lorentzian manifold with a semi-symmetric metric connection, Publ.Math.Debrecen, 104(3-4):, 2024, 329-321.