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GEOMETRIC AND VARIATIONAL ANALYSIS OF HYPERELASTIC STRIPS WITH TIMELIKE BASE CURVES

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ABSTRACT

The developable ruled surfaces whose modified Darboux vector is determined according to the unit tangent vector field and the binormal vector field of the reference curve are known as rectifying strips. The elastic behavior of rectifying strips is studied by minimizing the functional defined by Sadowsky in 1930. A necessary condition for a rectifying strip to be elastic is that the reference curve is a critical point of the Sadowsk functional. The fact that the Sadowsky functional is proportional to the Wilmore functional provides a different perspective on Willmore surfaces, which are frequently studied in geometry. Recent studies have shown that the p-Willmore functional, which is a more general case of the Willmore functional and is known to have applications in biology and quantum mechanics, and the Sadowsky-type functional, which is a more general case of the Sadowsky functional, are proportional. Also, according to recent studies, the rectifying strips formed by the critical points of the Sadowsky type functional are introduced as hyperelastic strips (or p-elastic strips). It is noteworthy that the Euler Lagrange equations characterizing hyperelastic strips are used to determine geometric conservation laws with the help of conservative quantities defined with the help of Euclidean motions. In this paper, we define the modified Sadowsky-type functional in three-dimensional Minkowski space and using timelike reference curves, derive the Euler Lagrange equations that characterize the critical points of the modified Sadowsky-type functional. We denote hyperelastic strips with timelike base curves (or p-elastic strips with timelike base curves) as the rectifying strips whose base curves are solutions for these Euler Lagrange equations. We establish a connection between hyperelastic curves and hyperelastic strips with timelike base curves in the case of torsion-free. We obtain conserved quantities for hyperelastic strips with timelike base curves for Minkowski spaces with Poincaré isometry groups. Then, we get the first and second conservation laws of hyperelastic strips with timelike base curves.

***K*eywords** Conservation laws · Hyperelastic strips with timelike base curves · Sadowsky-type functional · Variational calculus.

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