VII International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2026)

ICMASE 2026

A HYBRID OF THE CGS METHOD AND THE GPBI-CGSTAB(L) METHOD FOR NONSYMMETRIC LINEAR SYSTEMS

Zhixuan Xu[1,*], Tomohiro Sogabe[1], Ronan Dupont[1], Shao-Liang Zhang[1]

[1]*Department of Applied Physics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8603, Japan.*

ABSTRACT

We focus on numerical methods for nonsymmetric linear systems, in particular on a hybrid variant developed within the framework of product-type Krylov subspace methods [3]. In [2], a hybrid variant of the CGS method and the GPBi-CG method was recently proposed for solving nonsymmetric linear systems. In this talk, we further investigate the possibility of this hybrid variant together with the GPBi-CGstab(L) method, which is a unified algorithm of the GPBi-CG method and the Bi-CGSTAB(ℓ) method [1].

Similar to the approach in [2], we consider a three-phase algorithm in which either the CGS method or the GPBi-CGstab(L) method is applied in each phase. The transitions between phases are controlled by user-defined parameters ϵ_1, ϵ_2. In the first phase, the GPBi-CGstab(L) method is applied. Then, when the relative residual falls below ϵ_1, the algorithm transitions to the second phase, in which the CGS method is employed. Finally, when the relative residual falls below ϵ_2, the algorithm transitions to the third phase, where the GPBi-CGstab(L) method is applied again. To switch to a different method, it can be easily achieved by changing the parameters ζ, η in the framework proposed in [3], while preserving the Krylov subspace information gained in the previous iterations.

Numerical experiments are conducted on three-dimensional convection-diffusion-reaction equations in both diffusion-dominant and convection-dominant settings. The results show that, in the convection-dominant setting, classic stabilized methods such as Bi-CGSTAB and GPBi-CG either break down or stagnate. Although the CGS method converges, it suffers from a significant loss of accuracy. In contrast, our porposed hybrid method achieves fast and robust convergence without a significant loss of accuracy.

Acknowledgement

This work was supported by JST (Moonshot R&D Program) Japan Grant Number JPMJMS24A3.

***K*eywords** Nonsymmetric linear systems · Krylov subspace methods · CGS · GPBi-CG · GPBiCGstab(L)

References

[1] K. Aihara, GPBi-CGstab(L), A Lanczos-type product method unifying Bi-CGstab(L) and GPBi-CG, Numer. Linear Algebra Appl. **27**(2020), e2298.

[2] T. Sogabe, S.-L. Zhang, GPBi-CG revisited: a hybrid of the CGS method and the GPBi-CG method for nonsymmetric linear systems, Japan J. Indust. Appl. Math., **42**, (2025), 1161–1175.

[3] S.-L. Zhang, GPBi-CG: generalized product-type methods based on Bi-CG for solving nonsymmetric linear systems, SIAM J. Sci. Comput. **18**(1997), 537–551.

[*]*Corresponding Author's E-mail: z-xu@na.nuap.nagoya-u.ac.jp*