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A PERMANENT REPRESENTATION OF PERRIN NUMBERS VIA A CERTAIN COMPLEX PENTADIAGONAL MATRIX

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ABSTRACT

Permanents play an important role in various branches of mathematics and applied sciences, including graph theory, combinatorics, physics, chemistry, electrical engineering, and computational mathematics. In particular, establishing relationships between permanents of structured matrices and well-known integer sequences has attracted considerable interest, since such representations provide elegant matrix interpretations together with effective computational techniques for recursive sequences [1, 2]. In recent years, several studies have investigated determinant and permanent representations of Fibonacci-, Pell-, and Perrin-type sequences by means of Hessenberg and related structured matrices, revealing remarkable connections between matrix theory and number theory [3, 4, 5].

Among third-order linear recurrence sequences, the Perrin sequence possesses remarkable algebraic, combinatorial, and number-theoretic properties and has applications in recurrence theory and primality testing [6]. The Perrin sequence $\{R_n\}$ is defined by

$$R_n = R_{n-2} + R_{n-3}, \qquad n \geq 3,$$

with initial conditions $R_0 = 3$, $R_1 = 0$, and $R_2 = 2$.

Motivated by previous matrix representations of Perrin numbers, this paper introduces a new class of complex pentadiagonal matrices and investigates their permanents. We prove that the permanent of the proposed complex pentadiagonal matrix coincides with the corresponding Perrin number, thereby establishing a new permanent representation of the Perrin sequence. The proof is based on the recursive expansion formula for permanents together with the special structure of the proposed matrix. Furthermore, a Maple procedure is developed to verify the theoretical results and to compute the permanents efficiently for matrices of arbitrary order.

The proposed construction extends the existing literature by introducing a complex pentadiagonal matrix representation for Perrin numbers, whereas previous studies have mainly focused on Hessenberg matrices. Consequently, the obtained results provide an alternative and computationally efficient framework for studying Perrin numbers and further strengthen the relationship between permanents of structured matrices and third-order linear recurrence sequences.

***K*eywords** Perrin numbers · permanent · pentadiagonal matrix · integer sequences

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