ON SQUARES IN THIRD ORDER LINEAR RECURRENCES

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ABSTRACT

In numbers theory, to find perfect powers in recursive sequences is very popular and historical topic. For example, $F_1 = F_2 = 1$, $F_{12} = 144$ and $L_1 = 1$, $L_3 = 4$ are the squares in Fibonacci $\{F_n\}$ and Lucas $\{L_n\}$ sequences ([1, 2]). In 1996, Pethő [6] proposed the following problem at 7^{th} International Research Conference on Fibonacci numbers and Their Applications

Are the only squares $T_0 = T_1 = 0,\ T_2 = T_3 = 1,\ T_5 = 4,\ T_{10} = 81,\ T_{16} = 3136 = 56^2$ *and* $T_{18} = 10609 = 103^2$ *among the number* T_n?

This problem is still unsolved. In this talk, we show that, under several conditions, 1 is only square in the sequence $\{S_n\}$ where $S_n = S_{n-1} + S_{n-2} + S_{n-2}$ with $S_0 = 3, S_1 = 1, S_2 = 3$ for $n \geq 0$.

***K*eywords** Squares, Tribonacci-Lucas numbers.

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