

PERTURBED MODULAR HAUSDORFF INTERPOLATIVE IFS AND FRACTAL ATTRACTOR STABILITY

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ABSTRACT

This study presents a modular Hausdorff interpolative approach to fractal attractors. In contrast to the classical Hutchinson construction, the ordinary Hausdorff distance is replaced by a scale-dependent modular Hausdorff distance. For a finite family of modular contraction mappings $T_1, T_2, \dots, T_N$, we study the modular Hutchinson operator

$$\mathcal{F}(A) = \bigcup_{i=1}^N T_i(A).$$

Under appropriate completeness and contractive assumptions, we prove the existence and uniqueness of a compact attractor $\mathcal{A}^*$ that satisfies $\mathcal{F}(\mathcal{A}^*) = \mathcal{A}^*$. Moreover, the Picard iteration generated by $\mathcal{F}$ converges to this attractor for every admissible initial compact set.

The second part of the study deals with perturbation stability. We explain that small modular Hausdorff perturbations of the Hutchinson operator lead to controlled deviations between the corresponding attractors. This provides a scale-dependent stability estimate for fractal generation and a modular extension of classical fractal attractor theory. A Sierpiński-type modular construction is included to illustrate the applicability of the result.

Keywords modular metric space · Hausdorff interpolative metric · Hutchinson operator · iterated function system · fractal attractor · stability

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References

- [1] Barnsley, M. F. and Rising, H. (1993). *Fractals Everywhere*. 2nd ed., Academic Press, Boston.
- [2] Büyükkaya, A. and Öztürk, M. (2024). Multivalued Sehgal-Proinov type contraction mappings involving rational terms in modular metric spaces, *Filomat*, 38(10), 3563–3576.
- [3] Chistyakov, V. V. (2010). Modular metric spaces, I: Basic concepts. *Nonlinear Analysis*, 72(1), 1–14.
- [4] Din, M. and Sintunavarat, W. (2026). On a first investigation of Hausdorff interpolative metric spaces with applications to fractal geometry and market equilibrium, *Results in Control and Optimization*, 23, 100722.
- [5] Hutchinson, J. E. (1981). Fractals and self-similarity, *Indiana University Mathematics Journal*, 30(5), 713–747.
- [6] Karapınar, E. (2024). On interpolative metric spaces, *Filomat*, 38(22), 7729–7734.

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