
A FRACTIONAL-ORDER MODELING FRAMEWORK AND OPTIMAL CONTROL SYNTHESIS FOR RAPE AND VIOLENT BEHAVIOR DYNAMICS

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ABSTRACT

This study presents a fractional-order mathematical model to investigate the dynamics of rape and violence within a human population. The work addresses the limitations of classical integer-order models, which often fail to capture memory effects, delayed behavioural responses, and the long-term psychological consequences associated with trauma and recovery. To incorporate these effects, the model is formulated using the Atangana-Baleanu-Caputo fractional derivative. The total population is divided into nine interacting compartments representing safe individuals, violent persons, individuals undergoing rehabilitation, vulnerable populations, rape victims, treatment and counseling processes, at-risk individuals, and active rapists or violent offenders. Analytical results establish that the model is mathematically well posed, with all state variables remaining nonnegative and bounded over time, ensuring biological feasibility. Both the rape-and-violence-free equilibrium and the endemic equilibrium are derived, providing conditions under which violent behaviours either die out or persist within the population. The basic reproduction number is obtained using the next-generation matrix approach and serves as a key threshold parameter. It is shown that when this value is less than unity, the rape-and-violence-free equilibrium is locally asymptotically stable, indicating the elimination of violent behaviour. Conversely, when it exceeds unity, the endemic equilibrium becomes stable, signifying the persistence of rape and violence. Sensitivity analysis reveals that recruitment into the violent and active rapist classes significantly drives the spread of violence, while rehabilitation and treatment efforts play a crucial role in reducing it. Numerical simulations are carried out in MATLAB using a predictor-corrector scheme for fractional orders ranging from 0.7 to 1.0. The results demonstrate that memory effects significantly influence the system dynamics by prolonging the persistence of violent and at-risk populations, delaying recovery processes, and extending rehabilitation and treatment durations compared to classical models. An optimal control framework is incorporated to evaluate intervention strategies, including education, economic empowerment, rehabilitation, treatment, and counseling. Simulation results show in the absence of control measures, violence and victimization persist over time. However, when optimal control strategies are applied, there is a substantial reduction in violent behaviour, vulnerability, and the number of rape victims. The findings also indicate that early, sustained, and combined interventions are more effective than isolated or reactive approaches. This study demonstrates that fractional-order modelling provides a more realistic and robust framework for understanding the complex dynamics of rape and violence. It also highlights the importance of integrated intervention strategies in reducing the long-term impact of violence, offering useful insights for policy formulation and resource allocation.

Keywords Fractional calculus · optimal control · Stability analysis · effective reproduction number · Violence and Rape

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