

## Social behaviors in both interacting species and the Allee effect. Their impact on a predation model.

## Virginia Belén GALLAR<sup>1,\*</sup>, Alejandro Maximiliano ROJAS-PALMA<sup>2</sup>, Marcelo Eduardo ALBERTO<sup>1,3</sup>, Eduardo Jaime GONZÁLEZ-OLIVARES<sup>4</sup>

<sup>1</sup>Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Cuyo, Mendoza, Argentina.
<sup>2</sup>Departamento de Matemática, Física y Estadística, Facultad de Ciencias Básicas, Universidad Católica del Maule, Talca, Chile.
<sup>3</sup>Facultad de Ciencias Agrarias, Universidad Nacional de Cuyo, Mendoza, Argentina.
<sup>4</sup>Pontificia Universidad Católica de Valparaíso, Chile.

## ABSTRACT

In a predation interaction between two species, considering different ecological phenomena can produce significant changes in the dynamics of the models that describe it. Among these phenomena, the use of physical refuges by prey as a defense strategy against the constant threat of predation stands out. This behavior, directly influenced by the perceived level of risk, alters the probability of encounter between predator and prey.

Another relevant phenomenon is intraspecific competition among predators, driven by different causes, such as space limitation, depletion of food, etc. As predator density increases, the fight for access to prey intensifies. The Allee effect on prey postulates that the per capita growth rate of prey is reduced for low population sizes. This can have significant consequences for population survival and permanence.

Although there are many other ecological phenomena, the study and analysis of a mathematical model that incorporates the three phenomena mentioned above is proposed, adding as an assumption that the fraction of the prey population that uses the refuge is directly proportional to the number of predators present in the environment.

The system of differential equations proposed will be proved as well-posed and feasible as a population model. This will involve demonstrating the boundedness and positivity of the solutions, and analyzing the repulsive nature of the point  $(\infty, 0)$ , ensuring that the populations do not evolve towards physically impossible values and that the system avoids non-biological behaviors.

In addition, the existence and stability of equilibrium points of the system will be studied, representing scenarios where the predator population dies out and the prey population reaches a steady status. To complement this analysis, numerical simulations showing some possible dynamic scenarios will be used to visualize the global behavior of the system solutions and how population densities vary over time, under different combinations of parameters and initial conditions.

**Keywords** Predator-prey model · Intra-specific competition among predators · Allee effect · Refuge · Stability

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<sup>\*</sup>Corresponding Author's E-mail: virgigallar@gmail.com

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