ICMASE 2025

A MALTHUSIAN PREDATOR-PREY MODEL WITH SATURATED FUNCTIONAL RESPONSE

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ABSTRACT

Mathematical models are widely used in ecology. Particularly in population ecology, predator-prey models have a long history since their proposal in the 1920's by mathematician Vitto Volterra, and there are numerous studies on them with in-depth analysis of their properties. The Gause-type of predator-prey models includes compartmentalized models in which biomass leaving the prey compartment enters the predator compartment with some eventual conversion efficiency. Several of these models consider ecological factors such as self-interference in prey or saturation of prey consumption by predators. One of them is the well-known Rosenzweig-MacArthur model, which incorporates the well-known logistic equation in the prey growth rate to include self-interference ecological factor in this compartment and a functional response to represent predator saturation factor. However, a type of predator-prey model that has been little studied in the literature is one that assumes unrestricted growth in prey. A Gause-type predation model with Malthusian prey growth and saturation in predators described by the Holling Type II functional response in hyperbolic form is proposed and studied in this work. The main results obtained are presented: the existence of a single positive equilibrium point (inside the first quadrant) that is unstable for all parameter values and the non-existence of limit cycles in the system. These results suggest that prey self-interference significantly influences the population dynamics predicted by the model, both in terms of the stability of equilibrium points and the existence of fluctuations in population size for both species. This assertion is based on a comparative study of the Rosenzweig-MacArthur model and the one presented in this paper.

Keywords Predator-prey model · malthusian growth · limit cycle · separatrix curve · stability

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