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HOPF BIFURCATION AND DELAY-INDUCED DYNAMICS IN HOLLING TYPE II RESPONSES

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ABSTRACT

Mathematical modeling of ecological interactions plays a crucial role in understanding population dynamics, species coexistence, and ecosystem stability. Among such models, trophic systems, especially those involving three species arranged in a food chain consisting of prey, an intermediate predator, and a top predator, serve as powerful frameworks for capturing complex biological phenomena. In this study, we analyze a delayed three-species food chain model proposed in [1], consisting of prey, an intermediate predator, and a top predator. Biological processes such as gestation, maturation, digestion, and behavioral responses are inherently delayed and do not occur instantaneously. To incorporate these time-dependent biological processes, we extend the original model by introducing two discrete time delays: one representing the time lag in prey capture and consumption by the intermediate predator, and the other accounting for the delay in nutrient assimilation by the top predator. Our aim is to investigate how these delays affect the stability and dynamical behavior of the system, with particular emphasis on the conditions leading to Hopf bifurcation. First, we nondimensionalize the system to reduce parameter complexity. Subsequently, we derive the characteristic equation and determine the critical delay value at which Hopf bifurcation occurs. Using center manifold theory and normal form reduction [2], the first Lyapunov coefficient will be calculated to determine the direction and stability of the bifurcating periodic solutions. Finally, numerical simulations are performed to verify the theoretical results and illustrate the dynamical behavior of the system at the critical delay value. Our analysis confirms that time delays can destabilize equilibria and induce oscillatory behavior, providing deeper insight into the dynamical mechanisms governing delayed trophic systems.

References

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