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A Complexity Analysis of an Infinity Norm Neighborhood Predictor-Corrector Linear Programming Version Algorithm

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Abstract

Predictor-Corrector methods are one of the most studied variants of interior-point methods, being Mehrotra's predictor-corrector algorithm for Linear Programming [2] one of the most successfully implemented, both in research works and commercial packages.
One way to advance in the investigation on interior-point methods is to obtain more efficient algorithms. A manner of achieving this goal is to search for methods with better complexity results; this is, methods with a lower upper bound on the time needed by an algorithm to solve a given instance. Several researches have devoted their attention to the study of the complexity of Linear Programming algorithms; namely, the works of Zhang e al. [3], [4], [5], [6], Dexter et al. [7] and Benhadid et al. [8], that proved the polynomial complexity of some Linear Programming interior-point method variants.
In [1], Bastos and Paixão presented some Linear Programming interior point approaches, directly targeted to some linear allocation problems, specifically transportation and assignment, where the special structure of the constraint matrix of these problems is used to simplify the computations. The current work focus on one of these methodologies, namely the primal-dual predictor-corrector one. In particular, a feasible variant of this algorithm, originally proposed for structured problems such as transportation and assignment problems, is specifically adapted to use the infinity norm neighborhood and, then, analyzed.
The infinity norm is an alternative to the 2-norm when computing the used neighborhood, as it is more scalable and allows for greater flexibility in individual component deviations, particularly regarding computational efficiency and local accuracy, while the 2-norm can be excessively restrictive in practice. Additionally, the infinity norm often defines a broader neighborhood, allowing the algorithm to take longer steps without violating feasibility constraints, and, thus, progressing faster in the initial iterations, particularly when still being far from optimality.
In this work, a complete theoretical analysis, that includes the derivation of step-size bounds, duality gap reduction, and the complexity bound is presented. It is demonstrated that the algorithm has polynomial complexity and Q-linear convergence is proved. To achieve the main result, some additional lemmas and propositions were developed and demonstrated. This process allowed to prove that the algorithm with the infinity norm neighborhood has iteration complexity of $\mathcal{O}(n^{1/2}\log(\frac{1}{\varepsilon}))$.

Keywords Feasible Predictor-Corrector Algorithm · Infinity Norm Neighborhood · Polynomial Complexity

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References

[1] Bastos, F., Paixão, J.: Interior-point approaches to the transportation and assignment problems on microcomputers, Investigação Operacional, vol.13, 1, 3–15 (1993)

[2] Wright, S.J.: Primal-Dual Interior-Point Methods, SIAM, Philadelphia, USA (1997)

[3] Zhang, Y.: On the convergence of a class of infeasible interior-point methods for the horizontal linear complementarity problem, SIAM Journal on Optimization, vol. 4, 1, 208-227 (1994)

[4] Zhang, Y., Zhang, D.: On polynomiality of the Mehrotra-type predictor-corrector interior-point algorithms, Mathematical Programming, 68, 303–318 (1995)

[5] Zhang, Z., Zhang, Y.: A Mehrotra-type predictor-corrector algorithm with polynomiality and Q-subquadratic convergence Annals of Operations Research, 62, 131 - 150 (1996)

[6] Liu, Liying and Hua, Tao: A polynomial interior-point algorithm with improved iteration bounds for linear optimization, Japan Journal of Industrial and Applied Mathematics, 41(2), 739–756 (2024)

[7] Dexter, Gregory and Chowdhury, Agniva and Avron, Haim and Drineas, Petros: On the convergence of inexact predictor-corrector methods for linear programming, International Conference on Machine Learning, , 5007–5038 (2022)

[8] Benhadid, Ayache and Merahi, Fateh: Complexity analysis of an interior-point algorithm for linear optimization based on a new parametric kernel function with a double barrier term, Numerical Algebra, Control and Optimization, 13 (2), 224–238 (2023)