Existence and Uniqueness of Solutions for Impulsive ψ-Caputo Fractional Boundary Value Problems

Cansel Kuyumcu[1,*], Erbil Çetin[2]

[1] *Department of Mathematics, Ege University, Izmir, Turkey*
[2] *Department of Mathematics, Ege University, Izmir, Turkey*

Abstract

Modern complex systems across diverse scientific and engineering domains require mathematical frameworks that capture both memory effects and sudden disruptions simultaneously. Classical differential equation approaches, while successful for many applications, have limitations when modeling systems that exhibit both hereditary properties and abrupt state changes. This recognition has motivated the development of mathematical frameworks combining fractional calculus with impulsive dynamics, creating powerful tools for modeling the memory dependence and discontinuous behaviors characteristic of many real-world phenomena.

The $\psi-$Caputo fractional derivative represents a significant advancement in fractional calculus by generalizing the classical Caputo operator through the introduction of a kernel function ψ. This generalization enables application-specific adaptation of memory characteristics, providing flexibility that has proven valuable across applications from biomedical engineering and epidemiology to finance and ecological management. The ψ-Caputo framework allows researchers to tailor memory effects to specific system behaviors, significantly expanding the modeling capabilities of fractional differential equations.

While substantial progress has been made for orders $\alpha \in (0, 1]$, higher-order systems with $\alpha \in (1, 2]$ present additional mathematical challenges that require sophisticated analytical approaches. The development of rigorous theory for these systems is particularly important because many physical phenomena naturally require $\alpha > 1$ to capture both strong memory and acceleration effects. Fractional diffusion-wave equations and vibrating systems with hereditary properties exemplify applications that inherently involve momentum effects requiring fractional orders greater than unity for proper mathematical representation.

This work establishes existence and uniqueness results for impulsive $\psi-$Caputo fractional differential equations of order $1 < \alpha \leq 2$ using fixed point theorems. Under appropriate conditions, we prove that these differential equations possess unique solutions. These theoretical results provide essential analytical foundations that complement the computational methods required for solving non-local fractional operators. Our contribution advances the mathematical theory while enabling enhanced modeling capabilities for systems requiring both complex memory effects and sudden state changes, with the flexible $\psi-$Caputo framework supporting application-specific adaptations across diverse scientific domains.

***K*eywords** Impulsive · ψ-Caputo Fractional Boundary Value Problem · Fractional Calculus · Fixed Point Theorem

References

[1] Almeida R., A Caputo fractional derivative of a function with respect to another function, Commun. Nonlinear Sci. Numer. Simul., 44: 460-481, 2017.

*Corresponding Author's E-mail: celinecanselkuyumcu@gmail.com

[2] Asma, Ali A., Shah K., and Baleanu D., Ulam–Hyers stability analysis to a class of nonlinear implicit impulsive fractional differential equations with three point boundary conditions, Adv. Differ. Equ., 2019: 1-27, 2019.

[3] Benchohra M. and Slimani B. A., Existence and uniqueness of solutions to impulsive fractional differential equations, Electron. J. Differ. Equ., 2009: 1-11, 2009.

[4] Kuyumcu C. and Çetin E., On Existence of Symmetric Positive Solutions for SSBVPs on Time Scales, J. New Theory, 2021: 86-98, 2021.

[5] Poovarasan R., Samei M. E., and Govindaraj V., Study of three-point impulsive boundary value problems governed by ψ-Caputo fractional derivative, J. Appl. Math. Comput., 70: 3947-3983, 2024.

[6] Selvam A. P. and Govindaraj V., Controllability Results for ψ-Caputo Fractional Differential Systems with Impulsive Effects, Qual. Theory Dyn. Syst., 23: 166, 2024.