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STRUCTURALLY FINITE CLASSES OF ORDER-PRESERVING FUNCTIONS IN THREE-VALUED LOGIC

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ABSTRACT

In this paper, we study structurally finite classes of order-preserving functions in the framework of three-valued logic (3VL). Specifically, we analyze the class $F \subseteq P_3$, where P_3 is the set of all functions in 3VL. A function class F is defined as structurally finite if it can be generated from a finite set of base functions. We focus on unary and binary order-preserving functions and introduce the notation $F(n)$ to represent the subset of functions in F depending on n variables.

Our main result shows that the classes $M(1)_3$, $KM(2)$, $DM(2)$, and $M(2)$ form a hierarchical structure in the lattice of order-preserving maps. We prove that the structural finiteness of these classes ensures that every function in $M \subseteq P_3$ can be generated by functions that depend on at most two variables:

$$KM(2) = K \cap M(2), \quad DM(2) = D \cap M(2).$$

Additionally, the overall lattice structure of these function classes can be depicted as:

$$M \supseteq M^{(2)} \supseteq \{D, K\} \supseteq \{DM^{(2)}, KM^{(2)}\} \supseteq M_3^{(1)}.$$

The structural finiteness of these function classes has important applications, particularly in optimizing systems like cellular networks (LTE, 5G) and compiler optimization, where a finite basis of functions simplifies algorithmic processes. These findings generalize results from Boolean logic to 3VL, opening avenues for further research in multi-valued logic systems and their applications in areas such as network traffic control, resource allocation, and computational logic [1, 2, 3, 4, 5, 6].

***K*eywords** Three-valued logic · Order-preserving functions · Structural finiteness · Multi-valued logic · Cellular network optimization

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