DUMBBELL DYNAMICS AND TIDAL THEORY: THE CASE OF THE EARTH-MOON SYSTEM

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ABSTRACT

In this text, we address the modeling of gravitational tidal effects using the dumbbell model, widely recognized in celestial mechanics studies (4), (5), (6), (3). This model seeks to describe the dynamic behavior of elongated bodies interacting gravitationally with other objects in space, ideally representing tidal forces acting within such systems.The dumbbell model is particularly relevant for understanding the orbital and rotational evolution of celestial bodies, such as artificial and natural satellites, due to the tidal forces shaping their dynamics. In more recent studies, such as those presented in (2), dissipative terms have been incorporated into the equations of motion to account for energy dissipation caused by internal deformations of the bodies. These terms are crucial for explaining phenomena like the gradual deceleration of Earth's rotation due to oceanic tides and the stabilization of natural satellite orbits, such as the Moon's. The central aim of this text is to explore, initially, the classical results regarding the Earth-Moon system in the light of tidal theory and the dynamics of dumbbell-shaped systems, considering dissipative aspects.

***K*eywords** Tides, dumbbell model, orbital stability, Lagrangian formalism, ordinary differential equations.

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