
MATHEMATICAL ANALYSIS OF ALGAL DISEASES DYNAMICS WITH NUTRIENT-BASED OPTIMAL CONTROL STRATEGIES

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ABSTRACT

Harmful algal blooms (HABs) pose serious ecological and public health risks in aquatic ecosystems worldwide. In this research, we aim to investigate the complex dynamics of HABs by explicitly incorporating nutrient interactions into mathematical models. We propose a system of nonlinear differential equations that captures the interplay among algal populations, nutrient concentrations—primarily nitrogen and phosphorus—and disease-inducing agents such as algicidal bacteria or viruses. Analytical and numerical exploration of the model reveals rich dynamical behaviors, including oscillations, bistability, and threshold effects, all strongly influenced by nutrient availability and microbial activity. Building on this framework, we apply optimal control theory to design eco-friendly strategies for bloom mitigation. Specifically, we examine approaches such as nutrient limitation, biomanipulation, and feedback control systems, which demonstrate promise in reducing bloom intensity while minimizing environmental impact. Thus, our study integrates rigorous mathematical analysis with practical management strategies, underscoring the critical role of nutrient-based interventions and highlighting the potential of mathematical tools to inform sustainable aquatic ecosystem management.

Keywords Harmful algal blooms (HABs) · Mathematical model · Equilibria and stability · Optimal control · Numerical simulations.

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