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Golden Ratio-Based Two-Step Subgradient Extragradient Methods for Variational Inequalities: Convergence Analysis and Applications in Signal and Image Processing

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Abstract

We propose two novel inertial algorithms for solving variational inequality problems in Hilbert spaces. The first method integrates the golden ratio technique into a two-step subgradient extragradient framework, achieving weak convergence under monotonicity and R-linear convergence under strong monotonicity. The second algorithm modifies this approach to ensure strong convergence of the generated sequence. Both methods employ adaptive step sizes, eliminating the need for Lipschitz constant estimation or line search. Numerical experiments demonstrate their competitive performance compared to existing techniques. Additionally, applications in signal recovery and image restoration highlight their practical superiority over state-of-the-art methods.

Keywords Golden Ratio Technique · Inertial Two-subgradient Extragradient Method · Variational Inequality Problems · Monotone VIPs · Self-adaptive Step Sizes · Image Restoration · Optimization

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