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THE HOMOTOPY PERTURBATION METHOD AND NON-LOCAL SOLUTION OF INTEGRO-DIFFERENTIAL EQUATION

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ABSTRACT

Many researchers have focused on integral-differential equations due to their pivotal role in engineering, mathematical modeling, and physical phenomena, and how to solve them under local conditions. These equations have been addressed using various techniques, with some turning to transforming the systems to be solved into a system of integral equations, which can be solved using semi-analytical or numerical methods. This depends on the type of kernel used. However, in this research, the advanced-phase equation will be studied under non-local conditions. This type of modern study enables researchers to identify the strengths and weaknesses of a material before using it, as it reveals the genes and properties of the material being used. Therefore, in this work, we will attempt to study a non-linear integro-differential equation with a continuous kernel using analytical methods. The problem will also be studied by comparing solutions under local conditions with solutions under non-local conditions and studying the resulting error in each case.In this study we will consider the the phase-lag nonlinear integro-differential equation

$$\frac{\partial}{\partial t}\left[\Psi\left(x,\, t+p\right)-f(x,t)\right]=\lambda\, h(t)\int_0^1 k\left(x,\, y\right)\Psi^{\alpha}\left(y,\, t\right)\, dy,\quad \alpha=1,2,3,\ldots,N.(1) \tag{1}$$

under the non-local conditions

$$\Psi\left(x,\, p\right)=v_1\left(x,\, p\right)+v_2\left(x,\, t\right),\Psi\left(x,\, 0\right)=v_3(x)+v_2(x,\, t),$$

$$\frac{\partial\,\Psi\left(x,\, 0\right)}{\partial\, t}=v_4\left(x\right)+\frac{\partial}{\partial\, t}v_2\left(x,\, t\right).$$

Here, $f\left(x,t\right)$ is a free term, $h\left(t\right)$ is a continuous function of time, λ is a parameter and $k\left(x,y\right)$ is a known continuous function that represents the kernel of position. The unknown function that has to be determined is $\Psi\left(u,\, t\right)$. Where, $\Psi^{\alpha}\left(u,\, t\right)$ represents a nonlinear term in the equation. Under considering Taylor's expansion,

$$\Psi(x,\, t+p)=\Psi\left(x,\, t\right)+\frac{p}{1!}\frac{\partial\,\Psi\left(x,\, t\right)}{\partial\, t}+\frac{p^2}{2!}\frac{\partial^2\,\Psi\left(x,\, t\right)}{\partial\, t^2}+\ldots,$$

A nonlinear mixed integral equation of the second kind with a continuous kernel will be created from equation (1). In order to solve the resulting integral equation, the Homotopy approach will be used in this search. We shall talk about several significant theorems. Applications' numerical solutions will be offered to confirm the effectiveness of the strategies discussed.

Keywords Integro partial differential equation · Mixed integral equation · phase-lag problem

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