ON LEONARDO NUMBERS AND THEIR MATRIX REPRESENTATION

Samet ARPACI[1,*], Fatih YILMAZ[1]

[1]*Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, Turkiye*

ABSTRACT

In this paper, we introduce and investigate a Hessenberg matrix T_n, which incorporates both local interactions of tridiagonal matrices and fixed-distance non-local couplings, thereby exhibiting a hybrid Hessenberg–tridiagonal structure. We establish a recurrence relation for its determinant and demonstrate that $\det(T_n)$ coincides with the Leonardo numbers, with further connections to Fibonacci numbers and Chebyshev polynomials. The characteristic polynomial of T_n is derived through a Schur complement approach, and its spectral properties are examined in detail. The eigenvalue distribution reveals non-Hermitian features, including the presence of complex conjugate pairs and oscillatory instabilities. Moreover, an explicit LU factorization is obtained, enabling efficient computation of determinants and shedding light on the sparse structural properties of the matrix. Numerical experiments and visualization of eigenvalue behavior confirm the theoretical findings and highlight the potential of T_n as a model for structured non-Hermitian operators relevant in dynamical systems, wave propagation, and transport phenomena.

Keywords Hessenberg matrix · Determinant formulas · Leonardo numbers · Chebyshev polynomials · Eigenvalue analysis · LU decomposition

References

[1] Qi, F., & Kouba, O. (2017). A determinant of a generalized Hessenberg matrix and its application. Mathematica Slovaca, 67(6), 1403–1410. https://doi.org/10.1515/ms-2017-0445

[2] Leerawat, U., & Daowsud, K. (2022). On the determinants of some lower Hessenberg matrices. Special Matrices, 10(1), 132–142. https://doi.org/10.1515/spma-2022-0170

[3] Goy, T., & Shattuck, M. (2023). Determinants of Toeplitz–Hessenberg matrices with generalized Leonardo number entries. Annales Mathematicae Silesianae, 37(1), 73–91. https://doi.org/10.31261/amsil.16728

[4] Mirashe, N., Moghaddamfar, A. R., & Mozafari, S. H. (2010). The determinants of matrices constructed by subdiagonal, main diagonal and superdiagonal. Applied and Computational Mathematics, 9(3), 311–317. https://doi.org/10.1134/S1995080210030133

[5] Merca, M. (2013). A note on the determinant of a Toeplitz–Hessenberg matrix. Special Matrices, 1, 10–16.

[6] Goy, T., & Shattuck, M. (2019). Determinant formulas of some Toeplitz–Hessenberg matrices with Catalan entries. Proceedings – Mathematical Sciences, 129(46). https://doi.org/10.1007/s12044-019-0513-9

[7] Goy, T., & Shattuck, M. (2023). Determinants of some Hessenberg–Toeplitz matrices with Motzkin number entries. Journal of Integer Sequences, 26(23.3.4).

[8] Słowik, R. (2018). Inverses and determinants of Toeplitz–Hessenberg matrices. Taiwanese Journal of Mathematics, 22(4), 901–908. https://doi.org/10.11650/tjm/180103

[*] *Corresponding Author's E-mail: samet.arpaci@hbv.edu.tr*

[9] Higham, N. J. (2023). What is a Hessenberg matrix? Retrieved from https://nhigham.com/2023/12/05/what-is-a-hessenberg-matrix/

[10] T. R. Vishnu and D. Roy. Heat transport through an open coupled scalar field theory hosting stability-to instability transition. Journal of Statistical Physics, 2024. arXiv:2402.04986.

[11] F. M. Fernández. On a class of non-Hermitian Hamiltonians with tridiagonal matrix representation. Annals of Physics, 443:169008, 2022. arXiv:2109.14540.