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$\mathcal{A}$-CHARACTER AND POINT-WISE COUNTABLE FAMILIES

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ABSTRACT

The general concept of a metrizable family of subspaces of a topological space was introduced and applied to the problem of selections of multivalued mapping by M. Choban in [1]. Then in a similar context the concept of a family of subspaces with first axiom of countability was introduced and studied by E. Mihaylova in [3]. Families of p-subspaces and families of $A(k)$-spaces were defined and studied by M. Choban and E. Mihaylova in [2].

All considered spaces are assumed to be Hausdorff.

Fix a non-empty family $\mathcal{A}$ of non-empty subspaces of a topological space X.
A subset H of X is called an $\mathcal{A}$-subset of X if the set H is non-empty, and $H \subseteq L$ for some $L \in \mathcal{A}$.
A subset H of X is called an $\mathcal{A}$-balanced subset of X if the set H is non-empty and $H \subseteq \cap\{L : L \in \mathcal{A}, H \cap L \neq \emptyset\}$.

Definition 1 *The family γ of open subsets of X is called an $\mathcal{A}$-base of the $\mathcal{A}$-subset H in X if $H \subseteq \cap\{W : W \in \gamma\}$ and for every point $x \in H$ and every open subset U of X which contains H there exist an open subset V of X and $W \in \gamma$ such that $x \in V$ and $L \cap W \subseteq U$ for every $L \in \mathcal{A}$ such that $L \cap V \neq \emptyset$. The $\mathcal{A}$-character $\chi_X(H, \mathcal{A})$ of the $\mathcal{A}$-subset H of the space X is the smallest cardinal of the form $|\gamma|$, where γ is an $\mathcal{A}$-base of H.*

Lemma 1 *Let H be a compact $\mathcal{A}$-subset of the space X. Then there exists an $\mathcal{A}$-base γ of H such that:*
1. $|\gamma| = \chi_X(H, \mathcal{A})$ and $W \cap W' \in \gamma$ for every $W, W' \in \gamma$;
2. For every open subset U of X which contains H there exist an open subset V of X and $W \in \gamma$ such that $H \subseteq V$ and $L \cap W \subseteq U$ for every $L \in \mathcal{A}$ such that $L \cap V \neq \emptyset$;
3. If $\chi_X(H, \mathcal{A})$ is the countable cardinal $\aleph_0$, then there exists γ such that $\gamma = \{W_n : n \in \mathbb{N}\}$ and $W_{n+1} \subseteq W_n$ for every $n \in \mathbb{N}$.

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Definition 2 *A family $\mathcal{A}$ of subspaces of a space X is of pointwise-countable type if for every point $x \in \cup \mathcal{A}$ there exists a compact $\mathcal{A}$-balanced subset F of X such that $x \in F$ and $\chi_X(F, \mathcal{A}) = \aleph_0$.*

Theorem 1 *Let $f : X \longrightarrow Y$ be a continuous open mapping of a space X onto a space Y, $\mathcal{A}$ be a family of subspaces of the space Y and $\mathcal{A}^{-1} = \{f^{-1}(L) : L \in \mathcal{A}\}$ be a family of pointwise-countable type of the space X. Then $\mathcal{A}$ is a family of pointwise-countable type of the space Y.*

Keywords Character of a subspace · Point countable type space · Metrizable family of subspaces

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