

A SPECIAL CASE OF $\oplus$ -G-SUPPLEMENTED MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R -module. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and V is minimal for this property, then V is called a supplement of U in M . V is a supplement of U in M if and only if $M = U + V$ and $U \cap V \ll V$. If every submodule of M has a supplement in M , then M is called a supplemented module. If every submodule of M has a supplement that is a direct summand in M , then M is called a $\oplus$ -supplemented module. Let M be a supplemented module. If every supplement submodule in M is a direct summand of M , then M is called a strongly $\oplus$ -supplemented module. Let M be an R -module and $N \leq M$. If $T = M$ for every $T \trianglelefteq M$ with $M = N + T$, then N is called a g -small (or g -superfluous) submodule of M and denoted by $N \ll_g M$. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_g V$, then V is called a g -supplement of U in M . If every submodule of M has a g -supplement in M , then M is called a g -supplemented module. If every submodule of M has a g -supplement that is a direct summand in M , then M is called a $\oplus$ - g -supplemented module. Let M be a g -supplemented module. If every g -supplement submodule in M is a direct summand of M , then M is called a strongly $\oplus$ - g -supplemented module. In this work, some new properties of strongly $\oplus$ - g -supplemented modules are investigated. It is clear that every strongly $\oplus$ - g -supplemented module is $\oplus$ - g -supplemented. But the converse of this statement is not true in general.

Keywords Modules · g -Small Submodules · Supplemented Modules · g -Supplemented Modules

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