
A HIGH-ORDER ACCURATE ALE-SBP NUMERICAL SCHEME TO MODEL EVAPORATION

Hanifa Hanif^{1,2,*}, Arnaud. G. Malan¹,

¹*InCFD Research Group, Department of Mechanical Engineering, University of Cape Town, Cape Town 7700, South Africa.*

²*Department of Mathematics, Sardar Bahadur Khan Women's University, Quetta, Pakistan.*

ABSTRACT

We present a high-order numerical method for the canonical Stefan and sucking interface evaporation problems [1]. The governing energy equation is formulated within the Arbitrary Lagrangian-Eulerian (ALE) framework [2], which provides a systematic way to handle evolving computational domains. To avoid element collapse, the moving mesh is mapped onto a fixed reference domain through a coordinate transformation [3]. The transformed model is discretized using the continuous Galerkin (CG) finite element method (FEM) within the Summation-By-Parts (SBP) framework [4]. The SBP construction ensures discrete stability by mimicking continuous integration-by-parts identities. Numerical experiments are carried out with polynomial orders ranging from 1 to 4, allowing systematic evaluation of convergence behavior, accuracy, and efficiency. These experiments demonstrate the importance of high-order discretizations in capturing interface dynamics with reduced computational cost. In addition, employing high-order, provably stable discretizations ensures that reliable solutions to evaporation problems can be obtained with very small errors. Beyond benchmarking, the proposed ALE-FEM-SBP scheme establishes a foundation for robust simulation of phase-change phenomena, offering a provably stable and high-order accurate approach that can be extended to more complex geometries and multiphysics settings.

Keywords Evaporation · Stefan problem · sucking problem · summation-by-parts · continuous Galerkin method

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References

- [1] V. Voller. *For solving phase change problems*. Advances in numerical heat transfer 1 (1996): 341.
- [2] H. Cyrill W, A. Amsden, and J. Cook. *An arbitrary Lagrangian-Eulerian computing method for all flow speeds*. Journal of computational physics 14.3 (1974): 227-253.
- [3] T. Dennis, and C. Lombard. *Geometric conservation law and its application to flow computations on moving grids*. AIAA journal 17.10 (1979): 1030-1037.
- [4] H. Hanif, J. Nordström, and A. Malan. *Efficiency analysis of continuous and discontinuous Galerkin finite element methods*. AIMS Mathematics, vol 10.9 (2025): 22579-22597.

*Corresponding Author's E-mail: hanifa.hanif@uct.ac.za