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INTUITIVE APPROXIMATIONS FOR A RESIDUAL WAITING TIME PROCESS

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ABSTRACT

Let $W(t) \equiv T_{N(t)} - t$ denote the residual waiting time at time t, where $T_{N(t)} = \sum_{i=1}^{N(t)} X_i$ is the last renewal epoch before or at time t, and $\{X_n\}_{n\geq 1}$ is a sequence of idependent and identically distributed positive random variables generating a renewal process $N(t)$. The residual waiting time process occupies a central position in the analysis of stochastic systems, serving as a fundamental component in reliability theory, queueing models, and inventory dynamics. Its ability to quantify the time remaining until the next event renders it indispensable in modeling delays and forecasting performance in real-time systems. Accurate computation of its expected value is particularly critical in applications such as service operations, transportation planning, and communication networks, where delays translate directly into costs or inefficiencies.

Although its importance is well-recognized, the analytical study of $W(t)$ remains challenging.

In this work, we derive intuitive approximation formulas for the expected value of the residual waiting time process under two specific distributional settings:
(i) When the interarrival times X_n follow a regularly varying distribution with index $\alpha > 2$,
(ii) When X_n belongs to the $\Gamma(g)$ distribution family.
Regularly varying distributions play a central role in extreme value theory due to their heavy-tailed nature and their connection to the Fréchet domain of attraction. However, the regularly varying case does not include examples such as the exponential distribution or the gamma distribution. In extreme value theory, the class of $\Gamma(g)$ appears in a natural way to cover such light- or intermediate-tailed cases, including generalized extreme value and logistic distributions. To achieve these results, we utilize intuitive approximation techniques for the renewal function developed by Mitov and Omey. To the best of our knowledge, the literature does not seem to provide intuitive approximations for the residual waiting time in these settings.

Keywords Residual waiting time process · Intuitive approximation · Renewal function · Regularly varying distributions · $\Gamma(g)$ class of distributions.

References

[1] Khaniyev T. A., About moments of generalized renewal process, *Trans. Natl. Acad. Sci. Azerb., Ser. Phys. Tech. Math. Sci.*, **25**(1): 95–100, 2005.

[2] Mitov V. and Omey E., Intuitive approximations for the renewal function, *Statist. Probab. Lett.*, **84**: 72–80, 2014.

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