BICOMPLEX MANDELBAR DYNAMICS ASSOCIATED WITH THREE CONJUGATIONS

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ABSTRACT

In this paper, we investigate the antiholomorphic counterpart of bicomplex quadratic dynamics by defining and analyzing bicomplex Mandelbar sets corresponding to the three natural involutive conjugations of the bicomplex algebra. For each conjugation $\dagger_m$, $m \in 1, 2, 3$, we study the iteration

$$F_{c,\dagger_m}(\eta) = (\eta^{\dagger_m})^2 + c$$

on $\mathbb{C}_2$ and define the associated parameter set via the boundedness of the orbit of the origin. Our main objective is to determine how the choice of bicomplex conjugation influences the structure of the resulting antiholomorphic parameter space. Using the idempotent decomposition of bicomplex numbers, we obtain a conjugation-dependent classification of the corresponding dynamics. The conjugation $\dagger_3$ yields a genuine Cartesian product of two classical Mandelbar sets, whereas $\dagger_2$ generates a coupled two-parameter quartic system. In contrast, $\dagger_1$ produces a conjugate-dependent coupling between the idempotent components and therefore does not admit a Cartesian-product representation. We also introduce three-dimensional slices of these fractal sets for visualizations and investigate their reflection symmetries. Theoretical results and graphics demonstrate that the three conjugations generate geometrically distinct fractal structures despite sharing certain symmetry properties. These results clarify the role of bicomplex conjugations in antiholomorphic dynamics and show that bicomplex Mandelbar theory exhibits structural phenomena that do not arise in the holomorphic bicomplex Mandelbrot setting.

Keywords Bicomplex numbers · Mandelbar set · antiholomorphic dynamics · idempotent decomposition · bicomplex conjugations · fractal visualization.

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