
ON DETERMINANT REPRESENTATIONS AND GENERATING FUNCTIONS FOR GENERALIZED AVICENNA NUMBERS

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ABSTRACT

In this talk, we revisit the generating function for the recently introduced generalized Avicenna numbers. Using the Hessenberg determinant representation for linear recurrence relations, we show that the rational generating function of this family of sequences follows from a general structural framework. We also consider a class of sequences defined by binomial-coefficient recurrences, which we refer to as Pascal row recurrences, and derive their generating functions in closed form. These generating functions have denominator $(1 - t)^r$, with an explicit numerator determined by transformed initial conditions.

Keywords Avicenna numbers · Generating functions · determinant · Pascal triangle · Hessenberg matrix

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