VII International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2026)

ICMASE 2026

REQUIRED NUMBER OF OBSERVATIONS FOR ACCURATE STATE ESTIMATION IN THE LORENZ96 MODEL UNDER DIFFERENT FORCING PARAMETERS

Tianyuan Zhao[1,*], Kota Takeda[1], Tomohiro Sogabe[1], Shao-Liang Zhang[1]

[1]*Department of Applied Physics, Graduate School of Engineering, Nagoya University*

ABSTRACT

Data assimilation is a method to improve the accuracy of state estimation by combining model predictions with observation data. However, observing all state variables is infeasible in many high-dimensional systems, such as those encountered in weather forecasting and ocean modeling applications. Consequently, for a given dynamical system, the accuracy of state estimation depends on the choice of the observation operator. Previous studies have established theoretical and numerical results for data assimilation in partially observed chaotic dynamical systems [1, 2]. However, conventional theoretical analyses often consider fixed observation operators that do not account for the time-varying instability of the underlying dynamics. To address this limitation, adaptive observation strategies have been proposed [3], which target the most unstable directions of the dynamics at each time step. These strategies can reduce the number of observations required for accurate state estimation. However, it remains unclear how their performance varies with the degree of instability of the underlying dynamics.

In this work, as a typical example of data assimilation for chaotic dynamical systems, we consider the three-dimensional variational (3DVAR) method applied to the Lorenz96 model [4]. Within this framework, we investigate adaptive-in-time observation strategies under different values of the forcing parameter F, which affects the instability and complexity of the dynamics. Numerical experiments are conducted to examine how the minimum number of observations required for accurate state estimation depends on the instability of the dynamics and the observation error. Our numerical results show that adaptive observations remain effective across a range of values of F, while the required number of observations increases with the instability of the dynamics. These results provide useful numerical evidence for understanding the performance of adaptive observation methods and may support future theoretical developments in adaptive observation strategies in data assimilation.

***K*eywords** Data assimilation · Chaotic system · Dynamical instability · 3DVAR · Adaptive observations · Filter accuracy

References

[1] Hayden K., Olson E., Titi E.S., Discrete data assimilation in the Lorenz and 2D Navier–Stokes equations, Physica D, 240: 1416–1425, 2011.

[2] Law K.J.H., Shukla A., Stuart A.M., Analysis of the 3DVAR filter for the partially observed Lorenz'63 model, Discrete Contin. Dyn. Syst., 34: 1061–1078, 2014.

[3] Law K.J.H., Sanz-Alonso D., Shukla A., Stuart A.M., Filter accuracy for the Lorenz 96 model: Fixed versus adaptive observation operators, Physica D, 325: 1–13, 2016.

[4] Lorenz E.N., Predictability: A problem partly solved, Seminar on Predictability, ECMWF, 1: 1–18, 1996.

[*] *Corresponding Author's E-mail: t-zhao@na.nuap.nagoya-u.ac.jp*