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FIXED POINT THEORY IN DIGITAL METRIC SPACES AND ITS APPLICATIONS TO IMAGE COMPRESSION: A UNIFIED FRAMEWORK

Aybala Sevde Özkapu[1,*], Özlem Acar[1],

[1] *Department of Mathematics, Faculty of Science, Selçuk University, 42003, Konya, Türkiye, and Selcuk University, Constructive Mathematical Analysis Research Laboratory (CMARL), Selcuklu, 42003, Konya, Türkiye*

Abstract

The intersection of fixed point theory, digital topology, and machine learning-oriented signal processing presents a compelling arena where classical mathematical tools are recast to address the discrete structures underlying digital imagery. This talk presents a unified account of three interrelated contributions that collectively advance the fixed point theory on digital metric spaces and its practical deployment in fractal image compression algorithms.

We begin by establishing new *common fixed point results* in digital metric spaces for a pair of commuting rational-type contraction mappings $\hat{S}$ and $\check{T}$ satisfying generalised contractive conditions involving five positive parameters. The theorem extends Jungck's commutativity framework to the digital setting, and its application to the Sierpiński triangle demonstrates how iterated function systems converge to self-similar attractors within this discrete topology [2].

The second contribution introduces *vector-valued θ-contractions* on digital metric spaces, where the distance function takes values in $\mathbb{R}^m$ equipped with the standard cone ordering. By lifting the scalar θ-contraction of Jleli and Samet to this multi-dimensional setting, we prove a fixed point theorem that simultaneously monitors pixel intensity fidelity (via Mean Squared Error) and structural edge preservation (via gradient MSE). A comparative study on the standard 256×256 `Astronaut` image shows that the resulting Digital θ-Contraction algorithm achieves superior Structural Similarity Index Measure (SSIM = 0.6755) relative to classical scalar benchmarks, owing to the partial-ordering check enforced over both metric components [1].

The third contribution proposes *generalised rational digital F-contractions* within a vector-valued framework, introducing the Adaptive Digital F-Fractal Compression (ADFFC) algorithm. The key novelty is the rational maximum functional

$$M(x,\tilde{y}) = \max\left\{d(x,\tilde{y}),\, d(x,Tx),\, d(\tilde{y},T\tilde{y}),\, \tfrac{1}{2}\left[d(x,T\tilde{y}) + d(\tilde{y},Tx)\right]\right\},$$

which broadens the classical Banach and Wardowski conditions and drives a recursive quadtree decomposition whose depth is governed by the tunable parameter τ. Experimental analysis over the range $\tau \in [0.001, 1000]$ identifies three operational regimes: an insensitivity range (very low τ, maximal compression), a transition range ($\tau \in [1.0, 10.0]$, steep quality improvement), and a saturation region ($\tau \gtrsim 10$, diminishing returns). At $\tau = 5.623$, the ADFFC method reaches a block count of 3277 and PSNR of 18.18 dB for grayscale data, striking a practical balance between compression ratio and image fidelity [3].

Taken together, these results demonstrate that the mathematics of fixed point iterations in discrete spaces constitutes a rigorous foundation for adaptive, interpretable compression algorithms — a line of inquiry directly relevant to the aims of COST Action CA24131 (ENRICH), which seeks to advance transparent, mathematically grounded frameworks for explainable artificial intelligence and complex modeling applications.

* *Corresponding Author's E-mail: aybalasevdeozkapu@yahoo.com*

Keywords Digital metric space · Fractal image compression · ADFFC algorithm

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