

RADICAL INJECTIVE MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R -module. Let M be an R -module and $N \leq M$. If there exists a submodule K of M such that $M = N + K$ and $N \cap K = 0$, then N is called a direct summand of M and it is denoted by $M = N \oplus K$. For any R -module M , we have $M = M \oplus 0$. Let M be an R -module. Let M be an R -module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N + L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. A proper submodule N of an R -module M is called a maximal submodule of M if there exist no proper submodules with $N \leq K \leq M$ and $K \neq N$. The radical of a module M is defined the intersection of all maximal submodules of M and denoted by $RadM$. If M have no maximal submodules then the radical of M is defined by $RadM = M$. Let M and N be two R -modules. M is said to be N -injective if every diagram

$$\begin{array}{ccccc} 0 & \longrightarrow & K & \xrightarrow{g} & N \\ & & \downarrow f & \swarrow & \\ & & M & & \end{array}$$

with exact row can be extended commutatively by an R -module homomorphism $N \longrightarrow M$. If M is T -injective for every R -module T , then M is called an injective R -module. Let M and N be two R -modules. N is said to be radical M -injective (briefly, r - M -injective) if for every exact sequence $0 \longrightarrow K \xrightarrow{g} M$ with $RadM \leq g(K)$ and every R -module homomorphism $f : K \longrightarrow N$, there exists an R -module homomorphism $h : M \longrightarrow N$ with $h \circ g = f$. If N is radical T -injective for every R -module T , then N is called a radical injective (briefly, r -injective) module. In this work, some properties of radical injective modules are investigated.

Keywords Modules · Small Submodules · Radical · Injective Modules

2020 Mathematics Subject Classification: 16D40, 16D50.

References

- [1] J. Clark, C. Lomp, N. Vanaja and R. Wisbauer, *Lifting Modules. Supplements and Projectivity In Module Theory*, Frontiers in Mathematics, Birkhauser, Basel, 2006.
- [2] C. Nebiyev and A. Pancar, Cofinitely Injective and Projective Modules, *Haceteppe Journal of Mathematics and Statistics*, 39 No. 2, 171-182 (2010).
- [3] D. W. Sharpe and P. Vamos, *Injective Modules*, Lectures in Pure Mathematics University of Sheffield, The Great Britain, 1972.
- [4] D. D. Yazıcı and C. Nebiyev, r -Injective Modules, Presented in *3rd International Antalya Scientific Research and Innovative Studies Congress*, Antalya-Turkey, (2023).
- [5] R. Wisbauer, *Foundations of Module and Ring Theory*, Gordon and Breach, Philadelphia, 1991.

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