GENERALISED TRIBONACCI HYBRID QUATERNIONS

Ferhat Kürüz[1,*], Ali Dağdeviren[2]

[1]*Department of Computer Engineering, Faculty of Engineering and Architecture, Istanbul Gelişim University, Turkiye*
[2]*Department of Computer Engineering, Faculty of Engineering, Khoja Akhmet Yassawi International Kazakh-Turkish University, Kazakhstan*

ABSTRACT

In the literature, many studies have been conducted on number sequences from the past to the present. Recently, the applications of these number sequences in different areas have accelerated. However, it has also been observed that there is an increase in the diversity of these number sequences. New sequences such as Leonardo, Narayana, Mersenne, Tribonacci and Nickel numbers have been added to these number sequences, which started with Fibonacci [3, 1, 7, 6, 2]. In this study, we will define generalized tribonacci hybrid quaternions using the Tribonacci sequence.

The generalised tribonacci sequence was first introduced in 1972 by Shannon and Horadam[8]. This sequence is the third-order recurrence relation, which is the most generalised form of numbers such as Tribonacci, Padovan-Perrin, Narayana, and 3rd-order Jacobsthal. The generalised tribonacci sequence is given as $\{V_n(V_0, V_1, V_2; r, s, t)\}$, where $r, s, t \in \mathbf{R}$ and $n \geq 0$. The n^{th} generalised tribonacci number is defined by the relation $V_n = rV_{n-1} + sV_{n-2} + tV_{n-3}$, [3]. The most well-known of these sequences are as follows:

- Tribonacci numbers $\{T_n(0, 0, 1; 1, 1, 1)\}$,
- Padovan-Perrin numbers $\{P_n(0, 1, 0; 0, 1, 1)\}$,
- 3. order Jacobsthal numbers $\{J_n(0, 1, 1; 1, 1, 2)\}$.

The nth generalised hybrid tribonacci number HV_n is defined by the relation

$$HV_n = V_n + V_{n+1}\mathbf{i} + V_{n+2}\varepsilon + V_{n+3}\mathbf{h},$$

where V_n is the nth generalised tribonacci number, and $\mathbf{i}, \varepsilon, \mathbf{h}$ are hybrid units [9]. Let HV_n be the generalised hybrid tribonacci number. Then after some necessary calculations, one can obtain the following recurrence relation:

$$HV_n = rHV_{n-1} + sHV_{n-2} + tHV_{n-3}, \quad n \geq 3$$

with initial conditions:

$$\begin{aligned} HV_0 &= a + b\mathbf{i} + c\varepsilon + (rc + sb + ta)\mathbf{h}, \\ HV_1 &= b + c\mathbf{i} + (rc + sb + ta)\varepsilon + \left(\left(r^2 + s\right)c + (rs + t)b + rsa\right)\mathbf{h}, \\ HV_2 &= c + (rc + sb + ta)\mathbf{i} + \left(\left(r^2 + s\right)c + (rs + t)b + rsa\right)\varepsilon \\ &\quad + \left(\left(r^3 + 2rs + t\right)c + \left(r^2 s + s^2 + rs\right)b + \left(r^2 s + st\right)a\right)\mathbf{h}. \end{aligned}$$

The generalised tribonacci quaternions is defined as

$$Q_{v,n} = V_n + V_{n+1}i + V_{n+2}j + V_{n+3}k, \quad n \geq 0.$$

where V_n is the n-th generalised tribonacci number [3]. Hybrid quaternions were introduced by Dağdeviren in [4] as a new number system. A hybrid quaternion is

$$HQ = Q_0 + \mathbf{i}Q_1 + \varepsilon Q_2 + \mathbf{h}Q_3$$

*Corresponding Author's E-mail: fkuruz@gelisim.edu.tr

such that Q_0, Q_1, Q_2 and Q_3 are quaternions. The other presentation of a hybrid quaternion is

$$HQ = H_0 + iH_1 + jH_2 + kH_3$$

such that H_0, H_1, H_2 and H_3 are hybrid numbers. Sum, subtraction and multiplication operations can be easily done with the information up to here. In the literature there are many other works about hybrid quaternions, we can refer ([5]). In this study, we first introduce a novel sequence associated with hybrid quaternions, referred to as the generalized Tribonacci hybrid quaternion, which has not been previously examined. Subsequently, we investigate the fundamental definitions and key properties of this newly defined sequence. Additionally, we present the addition operation, derive the Binet formula, and discuss various specific variants of the generalized Tribonacci quaternions. Finally, we describe some characteristics of particular types of generalized Tribonacci quaternions.

***K*eywords** Tribonacci numbers · Hybrid quaternion · Number sequences

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