ON THE ALGEBRAIC PROPERTIES OF FUZZY JACOBSTHAL NUMBERS

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ABSTRACT

In this paper, we investigate a novel fuzzy extension of the classical Jacobsthal number sequence, called fuzzy Jacobsthal numbers. By embedding the Jacobsthal recurrence relation into the framework of fuzzy set theory, each term of the sequence is represented as an interval-valued quantity parameterized by a fuzziness coefficient $\alpha \in [0, 1]$. This parameter regulates the uncertainty width of each term, such that the classical Jacobsthal sequence is recovered when $\alpha = 1$, whereas maximal uncertainty occurs for $\alpha = 0$. A series of new theoretical results are established. First, the well-definedness and structural consistency of the fuzzy intervals are rigorously proven. Second, the fuzzy Jacobsthal numbers are represented through matrix and interval companion matrix formulations, enabling algebraic manipulation within the framework of interval arithmetic. Third, the spectral characteristics of the associated interval matrices are analyzed to determine asymptotic behavior and stability conditions. The proposed model not only generalizes the Jacobsthal sequence into the fuzzy domain but also reveals a clear connection between fuzziness propagation and exponential growth dynamics. These results provide a systematic foundation for the development of fuzzy recurrence relations and open avenues for uncertainty quantification in discrete dynamical systems.

Keywords Fuzzy Jacobsthal numbers · Interval analysis · Matrix representation · Spectral radius · Stability theory · Uncertainty propagation

References

[1] R. E. Moore, *Interval Analysis*, Prentice-Hall, 1966.

[2] S. Markov, Interval matrices and their eigenvalues, *Linear Algebra and its Applications*, 1996, 233: 89–109.

[3] B. Nelik, S. Halici, On Fuzzy Jacobsthal and Jacobsthal-Lucas Numbers, *European Journal of Pure and Applied Mathematics*, 2026, Vol. 19, Issue 1, Article Number 6677.

[4] L. A. Zadeh, Fuzzy Sets, *Information and Control*, 1965, 8(3): 338–353.

[5] R. Goebel, R. G. Sanfelice, Hybrid Dynamical Systems: Modeling, Stability, and Robustness, *Princeton University Press*, 2012.

[6] L. A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[7] A. Kaufmann, *Introduction à la Théorie des Sous-ensembles Flous*, Masson, Paris, 1975.

[8] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.

[9] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall, New Jersey, 1995.

[10] R. E. Moore, *Interval Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1966.

[11] S. Markov, "Interval matrices and their eigenvalues," *Linear Algebra and Its Applications*, vol. 233, pp. 89–109, 1996.

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