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Generalized Convex B-metric Spaces and Enriched Rational Type Contractions on Quasi-Banach Spaces

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Abstract

Let X be a nonempty set and $T : X \to X$ be a self mapping. If there exists a point $x \in X$ such that $Tx = x$, then x is called a fixed point of T. Banach [1] gave the famous Banach contraction principle as follows:

"Let (X, d) be a complete metric space and T be a self-mapping on X. If there exists $\kappa \in [0, 1)$ such that $d(Tx, Ty) \leq \kappa d(x, y)$, for all $x, y \in X$. Then, T has a unique fixed point. Furthermore, the Picard iteration $\{x_n\}$ defined by $x_n = Tx_{n-1}$, for all $n \in \mathbb{N}$, converges to the fixed point of T."

Banach contraction principle has been generalized by many researchers by revising the topology of the space or the contractive condition of the mapping T. With Berinde's [2] introduction of enrichment concept on Banach spaces, fixed point results for enriched contraction mappings became popular for researchers. In [2], enriched contraction introduced as follows:

Let $(X, \|.\|)$ be a linear normed space. A mapping $T : X \to X$ is said to be an enriched contraction if there exists $b \in [0, \infty)$ and $\theta \in [0, b + 1)$ such that

$$\| b(x - y) + Tx - Ty \| \leq \theta \| x - y \| \text{ for all } x, y \in X. \tag{1}$$

Moreover, they proved some fixed point theorems for Banach spaces by using Krasnoselskij iteration given by

$$x_{n+1} = (1 - \lambda)x_n + \lambda Tx_n, n \in \mathbb{N} \tag{2}$$

B-metric space has been introduced by Bakhtin [3] and researchers obtained many fixed point results in b-metric spaces. In [4], convex b-metric space was introduced and some fixed point theorems were extended to this new concept. Quasi-norm, which is a very related concept with the b-metric spaces, was given by Hyers [7]. Recently, Berinde [5] proved some fixed point results for enriched contraction mapping in quasi-norm spaces which are the generalizations of many enrichment results. In this work, we introduce the enriched rational type contraction mapping which is an extension of the contraction mapping given in (1) and we present some fixed point theorems for enriched rational type contractions in quasi-norm spaces and we obtain an extension of Dass-Gupta rational type fixed point result given in [6] to the enrichment concept by using Krasnoselskij iteration procedure. Also, we examine weak enriched rational type contraction by Kirk's iteration. Morever, by introducing the notion of generalized convex b-metric space, we give some fixed point results in generalized convex b-metric spaces.

Keywords Fixed point · Enriched rational type contraction · Quasi-norm

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