Analytical Investigation of Nonlinear Compressible Flow Using Lie Symmetry and Conservation Laws

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Abstract

Nonlinear compressible flow models play a significant role in describing wave propagation, shock formation, and energy transport in fluid dynamics and continuum mechanics. In this work, a comprehensive analytical study of the one-dimensional compressible flow equations is presented using Lie symmetry analysis. The governing nonlinear system, representing the conservation of mass, momentum, and energy, is investigated to determine its Lie point symmetries and the corresponding Lie algebra. An optimal system of one-dimensional subalgebras is constructed through the adjoint representation, enabling systematic similarity reductions of the governing partial differential equations. These reductions produce several exact invariant and travelling-wave solutions that describe different nonlinear flow behaviours. To enhance the physical interpretation of the model, nonlinear self-adjointness is established and employed to derive conservation laws associated with the transport of mass, momentum, and energy. Furthermore, modulation stability is examined through linear perturbation analysis. The resulting dispersion relation confirms the existence of stable convective and acoustic wave modes, indicating bounded propagation of small disturbances. The evolution of characteristic shock waves and weak discontinuities is also investigated to understand the behaviour of nonlinear wavefronts in compressible media. The obtained analytical solutions reveal important characteristics of density, velocity, and pressure distributions and provide valuable insight into nonlinear wave propagation. The proposed symmetry-based framework offers an efficient alternative to purely numerical approaches while preserving the underlying mathematical structure of the governing equations. The methodology can be extended to more complex nonlinear systems arising in mathematical physics, continuum mechanics, gas dynamics, and related engineering applications.

Keywords Lie symmetry analysis · Compressible flow · Invariant solutions · Weak discontinuity · Conservation laws · Modulation instability · Nonlinear wave propagation.

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