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# THREE STEP NUMERICAL OPTIMIZED TWO HYBRID POINTS METHOD FOR SOLVING FOURTH ORDER INITIAL VALUE PROBLEMS

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## ABSTRACT

This paper presents a three-step optimized two-hybrid points numerical method for the direct solution of fourth-order initial value problems arising in various applications in science, engineering, and applied mathematics. The proposed method is formulated using interpolation and collocation techniques to derive a continuous hybrid multistep scheme for solving fourth-order differential equations without necessarily transforming them into equivalent systems of first-order equations, thereby reducing computational complexity. The incorporation of optimized hybrid points within the three-step framework significantly improves the accuracy, efficiency, and stability of the method. Theoretical analysis of the developed scheme is carried out to establish its fundamental numerical properties, including consistency, order of accuracy, zero-stability, and convergence, all of which confirms that the method is efficient. Numerical problems of both stiff and non-stiff were solved using the method, and the numerical results obtained are compared with existing methods available in the literature. The comparative analysis shows that the proposed method achieves higher accuracy, faster convergence, and significantly reduced numerical errors. These results show that the developed three-step optimized two-hybrid points method provides an effective and efficient computational tool for the numerical solution of fourth-order initial value problems.

**Keywords** Hybrid · Collocation · Non-stif

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