ON GAUSSIAN GENERALIZED EDOUARD NUMBERS

Elen Viviani Pereira Spreafico[1,*], Eudes Antonio Costa[2], Paula Maria Machado Cruz Catarino[3]

[1] *Institute of Mathematics-Federal University of Mato Grosso do Sul-UFMS*
[2] *Departament of Mathematics-Federal University of Tocantins-UFT*
[3] *Departament of Mathematics-University of Trás-os-Montes e Alto Douro-UTAD*

ABSTRACT

In the literature, many studies have been devoted to Gaussian numbers whose components are taken from special integer sequences, such as Fibonacci, Lucas, Pell, and Jacobsthal, (see [1, 2, 3, 4, 5, 6, 8], and references therein). Gaussian numbers have an algebraic and geometric structure, but especially when they are elements of recurring sequences, such as Horadam's and Leonardo's numbers, they define complex generalizations of generalized Fibonacci numbers.

This numbering system, the Gaussian numbers, was introduced by Horadam in [4]. Consider the field of complex numbers, denoted by $(\mathbb{C}, +, \cdot)$. The set of complex numbers is defined as $\mathbb{C} = \{a + bi \mid a,\ b \in \mathbb{R} \text{ and } i^2 = -1\}$, where i is the complex unit. It was established that $(\mathbb{C}, +)$ and $(\mathbb{C}, \cdot)$ are abelian groups. A Gaussian number is a complex number $z = a + bi$, where a and b are integers, see [2].

Following the definition given in [5], the Gaussian Fibonacci sequence of numbers $\{GF_n\}_{n\geq 0}$ and the Gaussian Lucas sequence of numbers $\{GL_n\}_{n\geq 0}$ are defined by the recurrence relation $GF_{n+1} = GF_n + GF_{n-1}i$, where $GF_0 = i, GF_1 = 1$, and $GL_{n+1} = GL_n + GL_{n-1}i$, where $GL_0 = 2 - i, GL_1 = 1 + 2i$, respectively. Some identities involving Gaussian Fibonacci sequences are established in [2].

In this article, we introduce the generalized Gaussian Edouard numbers $\{GW_n\}_{n\geq 0}$ defined by the recurrence relation

$$GW_m = 7GW_{m-1} - 7GW_{m-2} + GW_{m-3},$$

with arbitrary initial conditions GW_0, GW_1, GW_2. We establish the recurrence relation, generating function and the Binet formula for this new sequence of numbers.

As particular cases, when $GW_0 = 0, GW_1 = 1, GW_2 = 7 - i$, we have the Gaussian Edouard numbers $\{GE_n\}_{n\geq 0}$, and by taking $GW_0 = 3, GW_1 = 7 - 3i, GW_2 = 35 - 7i$, we get the Gaussian Edouard–Lucas numbers $\{GK_n\}_{n\geq 0}$. The relations of these new particular sequences with the Gaussian Balancing and Gaussian Lucas-Balancing numbers (see [8]) are explored, and some new identities are provided.

***K*eywords** Generalized Edouard numbers · Generating function · Binet's formula · Identities

References

[1] Berzsenyi G., Gaussian Fibonacci numbers, Fibonacci Quart., 15(3): 233-236, 1977.

[2] Halıcı S., and Oz S., On Some Gaussian Pell and Pell–Lucas numbers, Ordu Uni., Science and. Technology Journal, 6(1): 8-18, 2016.

[3] Harman C.J., Complex Fibonacci numbers, Fibonacci Quart., 19(1): 82-86, 1981.

[4] Horadam A.F., Complex Fibonacci numbers and Fibonacci quaternions, The American Math. Monthly, 70(3): 289–291.

Corresponding Author's E-mail: elen.spreafico@ufms.br

[5] Jordan J.H., Gaussian Fibonacci and Lucas numbers, Fibonacci Quart., 3(4): 315-318, 1965.
[6] Kaya A., and Ozimamoğlu H., On a new class of the generalized Gauss k-Pell numbers and their polynomials, NNTDM, 28(4): 593-602, 2022.
[7] Soykan Y., Generalized Edouard numbers,IJAAMM , 3(9), 41-52, 2022.
[8] Tasci D., Gaussian balancing numbers and Gaussian Lucas-balancing numbers, J. Sci. Arts , 3(44) : 661–666, 2018.