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NUMERICAL SOLUTION OF DELAY DIFFERENTIAL EQUATION WITH BOUNDARY LAYER

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ABSTRACT

In this work, we consider a boundary value problem for the singularly perturbed convection diffusion problem for a type of delay differential equation. Differential equations with a small positive parameter multiplied by the highest derivative term are said to be singularly perturbed differential equations. Generally, the solutions of such equation exhibit multiscale phenomena. Within certain thin sub-regions of the domain, the scale of some derivatives is significantly larger than other derivatives. These thin regions where rapid change occurs are called boundary or interior layers. These equations appear frequently in mathematical problems in the sciences and engineering. For example, high Reynolds number flows in the fluid dynamics, electrical networks, reaction-diffusion processes, control theory, the equations governing flow in porous media, the drift-diffusion equation of semiconductor devices are modelled by these equations [3, 5].

Generally, solutions to such problems often involve a boundary or initial layer. The classical discretization methods to solve singularly perturbed problems do not work well and fail to give analytical solution when tends to zero. For this reason, it is necessary to develop suitable numerical methods that are uniformly convergent with respect to solve this type of problems [2, 3]. If the singularly perturbed differential equations also contain the delay term, they are called the singularly perturbed delay differential equations. Lange and Miura investigated the asymptotic properties of the boundary value problem for singularly perturbed delay differential equations [4]. We are motivated by this paper for the numerical solution of our work. There are few studies in the literature using different numerical methods to solve these equations [1, 2].

For the numerical solution of the problem, we present a fitted difference scheme on a layer adapted mesh and analyze the error estimates. We show that the scheme is first-order convergent with respect to the perturbation parameter in the discrete maximum norm. Furthermore, we give the numerical experiments which support the theoretical results.

Keywords Delay differential equation · Boundary layer · Finite difference method · Uniform convergence

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