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Novel Fixed Point Results via Hybrid-Interpolative Reich-Istrăţescu-Type Contractions in Parametric S-Metric Spaces

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Abstract

Fixed point theory has long been recognized as a cornerstone of modern mathematical analysis, particularly in the context of nonlinear analysis, topology, and applications to differential equations, dynamical systems, and optimization. Among its classical results, Brouwer's Fixed Point Theorem [1] and Banach's Contraction Principle [2] have laid a strong foundational framework. Banach's theorem, in particular, ensures the existence and uniqueness of fixed points for contraction mappings on complete metric spaces, providing the theoretical underpinning for many iterative and convergence-based solution methods in applied mathematics.

As the limitations of classical metric spaces became evident in modeling complex and uncertain systems, various generalizations of the metric structure have been introduced. These include but are not limited to G-metric spaces, b-metric spaces, and S-metric spaces. The concept of an S-metric space, proposed by Sedghi et al. [3], is a notable extension of the classical metric space. In an S-metric space, the distance function is defined on three variables rather than two, offering greater analytical flexibility and an alternative approach for studying convergence and fixed point behavior. The advent of parametric S-metric spaces has engendered a novel analytical environment, thereby enhancing our capacity to address fixed point problems in uncertain or imprecise contexts.

In parallel, the generalization of contraction conditions has also received significant attention in recent literature. Various generalizations such as Kannan [4], Chatterjea [5], Reich [6], and Istrăţescu [7] contractions have been proposed to relax the strict assumptions of Banach's original formulation. Building upon this line of work, Karapınar et al. [8] introduced the hybrid-interpolative Reich-Istrăţescu type contraction, a flexible, parameter-dependent contractive mapping that unifies and generalizes multiple existing contraction types.

This study aims to contribute to the expanding field of metric fixed point theory by investigating fixed point results within the framework of parametric S-metric spaces using the hybrid-interpolative Reich-Istrăţescu contraction. To this end, we adapt and redefine the contraction condition introduced by Karapınar et al. [8] to suit the structure of parametric S-metric spaces. We then establish several new fixed point theorems under this generalized setting. The validity and applicability of the theoretical findings are demonstrated through illustrative examples.

By unifying the generalized metric framework of S-metric spaces and parametric structures and the richness of hybrid contraction mappings, this research significantly extends the current scope of fixed point theory. The results not only generalize existing theorems but also pave the way for further investigations into applications where uncertainty and higher-order metric structures play a central role.

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Keywords Hybrid-interpolative Reich-Istra¸tescu-type contractions, parametric S-metric spaces, S-metric spaces

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