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GEOMETRY AND ENTANGLEMENT OF SUPER-QUBIT QUANTUM STATES

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ABSTRACT

We introduce the super-qubit quantum state, determined by superposition of the zero and the one super-particle states, which can be represented by points on the super-Bloch sphere. In contrast to the one qubit case, the one super-particle state is characterized by points in extended complex plain, equivalent to another super-Bloch sphere. Then, geometrically, the super-qubit quantum state is represented by two unit spheres, or the direct product of two Bloch spheres. By using the displacement operator, acting on the super-qubit state as the reference state, we construct the super-coherent states, becoming eigenstates of the super-annihilation operator, and characterized by three complex numbers, the displacement parameter and stereographic projections of two super-Bloch spheres. The states are fermion-boson entangled, and the concurrence of states is the product of two concurrences, corresponding to two Bloch spheres. We show geometrical meaning of concurrence as distance from point-state on the sphere to vertical axes - the radius of circle at horizontal plane through the point-state. Then, probabilities of collapse to the north pole state and to the south pole state are equal to half-distances from vertical coordinate of the state to corresponding points at the poles. For complimentary fermion number operator, we get the complimentary super-qubit state and corresponding super-coherent state, as eigenstate of transposed super-annihilation operator.

***K*eywords** qubit · super-qubit · super-coherent state · super-Bloch sphere · entanglement

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