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CHARACTER-THEORETIC APPROACHES TO DIFFERENCE SETS

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ABSTRACT

Let G be a finite abelian group of order v, and let $D \subseteq G$ with $|D| = k$. D is called a (v, k, λ)-difference set if the list of differences $\{d - d' \mid d, d' \in D, d \neq d'\}$ covers every nonzero element of G exactly λ times. Character theory provides a powerful algebraic method for investigating such sets, particularly through the evaluation of group character sums.

In this study, we explore the classical character-theoretic condition which states that $D \subseteq G$ is a (v, k, λ)-difference set if and only if for every non-principal character χ of G,

$$\left| \sum_{d \in D} \chi(d) \right|^2 = k - \lambda v.$$

Using this condition, we investigate known difference sets such as the Singer difference sets and difference sets arising from cyclotomic classes in finite fields $\mathbb{F}_q$, where $q \equiv 1 \pmod{n}$.

Furthermore, we apply group ring techniques to represent subsets of G as formal sums in $\mathbb{Z}[G]$, and express the difference set condition as:

$$DD^{(-1)} = k \cdot 1_G + \lambda(G - 1_G),$$

where $D^{(-1)} = \{d^{-1} \mid d \in D\}$ and 1_G is the identity of G. This formulation connects combinatorial properties to algebraic identities and facilitates computer-aided verification.

Our work emphasizes how character sums not only simplify verification of known constructions but also guide the discovery of new families of difference sets. The results suggest further applications in combinatorial design theory, coding theory, and finite geometry.

***K*eywords** character theory · difference sets · group rings

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