ICMASE 2024

ABSTRACT BOOK

V INTERNATIONAL CONFERENCE ON MATHEMATICS AND ITS APPLICATIONS IN SCIENCE AND ENGINEERING (ICMASE 2024)

16-18 SEPTEMBER 2024

ISEC COIMBRA / PORTUGAL

Preface

This abstract booklet includes the abstracts of the papers that have been presented at V International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2024) which was held in ISEC (Instituto Superior de Engenharia de Coimbra), Portugal between 16-18 September, 2024, via hybrid. The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Diferantial Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, and Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2024.

Prof. Dr. Deolinda RASTEIRO,

Prof. Dr. Fatih YILMAZ

Chairs, ICMASE 2024

V International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2024)

16-18 September 2024, ISEC (Instituto Superior de Engenharia de Coimbra)

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CONTENTS

INVITED TALKS

Invited Speakers

- María JESÚS SANTOS SANCHEZ, Salamanca University, (Spain)
  Title: Modelling Solar Energy through Mathematical Models
- Samuel BENGMARK, University of Gothenburg, (Sweden)
  Title: Math Learning Material For Refugee Camps and the Global South
- Natalia BEBIANO, University of Coimbra, (Portugal)
  Title: Direct and Inverse Spectral Problems: Theory, Algorithms and Applications

ICMASE 2024

V. International Conference on Mathematics and its Applications in Science and Engineering

(ICMASE 2024)

Title: Modeling solar energy through mathematics

Abstract: We live in a critical moment for humanity in which energy consumption is growing as the population grows, while fossil fuel resources are diminishing. It is time to bet on the search for renewable energy sources. Here arises, as researchers, the challenge of making these sources efficient and adapting them to the real demand. If we were able to harness all the solar energy that our planet receives, it would be enough to supply our current demand. And here we come into play engineers, physicists and of course! Mathematics. On the one hand, regarding the production of renewable electrical energy, one of the main lines of research that we develop at the Energy Optimization, Thermodynamics and Statistical Physics Group of the University of Salamanca is the simulation of concentrating solar thermal energy. Such as a central tower plant, CSP. Analyzing possible aspects to improve the efficiency of the subsystems involved. And, on the other hand, to try to adapt the production to the demand, to study different aspects of thermal storage. All these processes are simulated by means of different programs such as Mathematica, Payton, Matlab, etc. And, of course, mathematical tools such as analytical and numerical integrals, nonlinear equation solving, differential equation, interpolations, multiobjective optimization, etc., are used for this purpose. Let's see how mathematics becomes a magic wand that transforms this desire to harness the energy coming from the sun into a reality.

V. International Conference on Mathematics and its Applications in Science and Engineering

[ICMASE 2024]

Title: Math Learning Material For Refugee Camps and the Global South

Abstract: To support mathematics learning in under privileged environments, the charity foundation Akelius Foundation collaborates with the Department for Mathematical Sciences at Chalmers and the University of Gothenburg. Together, they have formed Akelius Math Learning Lab, where digital learning material is developed that will be freely available in several languages. The lab is staffed with material developers, mathematicians, and mathematics education researchers. The development is based on ideas from education research and is continuously researched. The lab started four months ago, so no material has been published yet. However, I will tell you about the challenges we see, the educational ideas that we base the material on, and the results from our first classroom trials.

ICMASE 2024

V. International Conference on Mathematics and its Applications in Science and Engineering

(ICMASE 2024)

Title: Direct and Inverse Spectral Problems: Theory, Algorithms and Applications

Abstract: In this talk, direct and inverse spectral problems with applications in science and engineering are discussed. Necessary and sufficient conditions for the existence of solution are given, and construction algorithms based on the obtained results are provided. The efficiency of the proposed algorithms is analyzed, and illustrative examples are presented.

CONTRIBUTED TALKS

GENERALIZED FRACTIONAL INTEGRAL INEQUALITIES VIA CONVEXITY

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ABSTRACT

Convexity is considered as one of the principal classes of different aspects of mathematics such as geometry and functional analysis. Applications of convex functions are found in various scientific fields, including mathematics and engineering. The well-known results, identified as integral inequalities, are formulated using the convexity. The study of these new inequalities - which link the classical inequalities with fractional inequalities - has been considered to be an active area of interest by many scholars. Thus, in this article, we present the generalized fractional inequalities through fractional operators involving generalized convexity. Using our reported results, we demonstrate their applications to special means.

***K*eywords** Convex functions · Fractional integrals · Integral inequality

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DISTRIBUTED FUSION ESTIMATION IN THE PRESENCE OF MEASUREMENT QUANTIZATION AND MIXED ATTACKS

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ABSTRACT

Over the past years, distributed fusion estimation algorithms have emerged as important tools in the field of networked systems, providing profound significance and practical utility in various domains (see, e.g., [1]–[3]). These algorithms play an essential role in processing information collected from different sensors or sources, to make estimations about the underlying system state or the signal of interest. One of the key advantages of distributed fusion estimation is its ability to efficiently handle large and decentralized networks. By distributing the computational burden across multiple nodes or agents within the network, these algorithms mitigate the challenges associated with centralized processing, such as computational complexity, communication bandwidth constraints, and vulnerability to single points of failure.

The consideration of quantized measurements in signal estimation problems is becoming a topic of great importance in many real-world scenarios, such as wireless sensor networks or communication systems, where the processing of continuous-valued data packets results in significant energy consumption and bandwidth usage. Quantization, which involves discretizing continuous measurements into a finite number of levels, offers a viable solution to alleviate these challenges by reducing the amount of data that must be processed. Despite the loss of information inherent in quantization, properly designed signal estimation algorithms can effectively recover accurate estimates from quantized measurements. In addition, incorporating quantization into signal estimation frameworks promotes the development of more efficient and robust algorithms that can operate effectively in resource-constrained environments (see [4] and references therein).

In today's interconnected world, where critical infrastructure, communication networks, and cyber-physical systems are increasingly vulnerable to malicious activity, understanding and mitigating the impact of eventual attacks is essential for safeguarding against potential disruptions or compromises. Random deception attacks involve adversaries injecting false information into the system, leading to erroneous estimations and eventually compromising the integrity of decision-making processes. On the other hand, denial-of-service (DoS) attacks aim to disrupt the normal operation of the system by overwhelming it with a flood of malicious requests or traffic, thereby affecting the availability and performance of estimation algorithms. Due to the limitations of existing algorithms in this scenario, the study of the estimation problem in networked systems exposed to different types of attacks has received great attention in the last few decades (see [5], [6] and references therein, among others).

This paper addresses the distributed fusion estimation problem of stochastic signals from quantized measurements with random parameter matrices and time-correlated additive noises. These measurements are assumed to be exposed to mixed network attacks, including both random deception attacks and denial-of-service (DoS) attacks and the stochastic nature of these attacks is aptly modeled through Bernoulli random variables. Using a covariance-based methodology and a prediction compensation strategy to counteract the random loss of information caused by DoS attacks, recursive algorithms are designed for the distributed fusion filtering and fixed-point smoothing problems.

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Numerical simulation results are presented to underscore the broad applicability of using random parameter matrices, which effectively cover different kinds of network-induced uncertainties and random failures, thus providing a more realistic representation of engineering environments. Furthermore, the obtained results validate the efficiency of the proposed estimation scheme and provide insights into the influence of random attack probabilities on the estimation accuracy.

The proposed algorithms contribute significantly to the advancement of signal processing and network security research, particularly in contexts involving quantized measurements, mixed uncertainties, and network attacks. This comprehensive investigation enhances the understanding and mitigation strategies in complex networked systems, facilitating advances in both theoretical research and practical implementations.

***K*eywords** Distributed fusion estimation · quantized measurements · mixed attacks · random parameter matrices · time-correlated noise

Funding Grant PID2021-124486NB-I00 funded by MICIU/AEI/ 10.13039/501100011033 and ERDF/EU. National High-end Foreign Experts Recruitment Plan of China under grant G2023012004L.

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ON BIPOLAR FUZZY SOFT HYPERLATTICES

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ABSTRACT

The paper we study covers several topics related to fuzzy sets and hyperlattices (HLs). Firstly, it begins by introducing the concept of fuzzy sets and how they differ from classical sets. Then goes on to discuss bipolar fuzzy (BF) sets, which extend the concept of fuzzy sets to define each element with a degree of positivity and a degree of negativity. Next, Fuzzy soft (FZS) sets, which combine the ideas of fuzzy sets and soft sets, are discussed. Soft sets are a generalization of classical sets that allow for elements to have degrees of membership. Fuzzy soft sets take this idea further by allowing for uncertainty in both the membership and non-membership of elements. The paper then turns its attention to HLs, which are a generalization of lattices that allow for the representation of more complex relationships between elements. Specifically, the paper focuses on fuzzy, soft hyperideals (HIDs) of HLs. HIDs are a generalization of the concept of ideals in algebra, and FZS HIDs allow for greater flexibility in the definition of these structures. The paper illustrates some examples of fuzzy soft HIDs in HLs. Finally, the paper introduces the concept of bipolar fuzzy soft (BFS) HIDs, which are a combination of HIDs and FZS HIDs in HLs.

***K*eywords** Fuzzy soft set · Soft hyperideal · Bipolar fuzzy set · Bipolar fuzzy soft set · Bipolar fuzzy soft hyperlattices

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2-ABSORBING IDEALS OF COMMUTATIVE SEMIRINGS WITH BIPOLAR FUZZY LOGIC

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ABSTRACT

This study aims to provide a comprehensive understanding of the inherent characteristics, attributes, and operational behaviors of 2-absorbing ideals and 2-absorbing primary ideals of commutative semirings within the framework of bipolar fuzzy logic. We begin by considering the algebraic structure of these ideals and their behavior under the logical conjunction ($\wedge$) and disjunction ($\vee$) operations.

In bipolar fuzzy sets, the range of the membership degree is $[-1, 1]$. An element has no bearing on the associated property if its membership degree is 0. If an element's membership degree is within the range of $(0, 1]$, it partially fulfills a hidden counter attribute. Conversely, if an element's membership degree falls between $[-1, 0]$, it partially meets a hidden counter characteristic. These degrees are denoted by $\mu : S \to [-1, 1]$, reflecting the bipolar nature of the membership function.

Through a detailed exploration of their significance in existing literature, we seek to uncover their distinctive traits and potential applications across various logics. Notably, the bipolar fuzzy logic framework introduces a nuanced perspective on the membership degrees of elements within these ideals, where the functions $\mu^+ : X \to [0, 1]$ and $\mu^- : X \to [-1, 0]$ denote the degrees of membership and of non-membership of each element $r \in X$ to set A, respectively, $0 \leq \mu^+(r) - \mu^-(r) \leq 1$ for all $r \in X$.

Additionally, the definition of the Cartesian product for these ideals, $I \times J = \{(i, j) \mid i \in I, j \in J\}$, facilitates a more profound examination of their interrelations. This product allows us to extend properties of single ideals to pairs of ideals, thus enriching our understanding of their interactions and dependencies, thereby paving the way for novel research avenues in this logic. Furthermore, the role of lattice theory in structuring these relationships cannot be overlooked, as the set of 2-absorbing ideals forms a lattice under inclusion.

Lastly, our study encompasses an analysis of these ideals under a semiring homomorphism. Let $\phi : S \to T$ be a semiring homomorphism. If I is a 2-absorbing ideal of S, we examine the conditions under which $\phi(I)$ remains a 2-absorbing ideal in T within the framework of bipolar fuzzy logic. This investigation reveals deeper insights into the preservation of ideal properties across semiring homomorphisms with bipolar fuzzy logic and underscores the robustness of an application of bipolar fuzzy logic to 2-absorbing ideals within different algebraic frameworks.

***K*eywords** 2-absorbing ideal · 2-absorbing primary ideal · 2-absorbing bipolar fuzzy ideal.

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SHANNON ENTROPY AND MIGRATING POPULATIONS

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ABSTRACT

Information theory is the area of mathematics that studies the quantification, storage and communication of digital information. In its genesis is the purpose of quantifying the existing information in random events. This is an area fundamentally established after the work of Harry Nyquist [3] and Ralph Hartley [1] in the 1920s, but mainly by Claude Shannon in the 1940s, in his impressive article *A Mathematical Theory of Communication* [5], situated at the intersection of several disciplines: probability theory, statistics, computer science, statistical mechanics, information engineering and electrical engineering. Applications of fundamental information theory topics include encoding information sources, compressing data (*e.g.*, for ZIP files), encoding channels, as well as error detection and correction. The key concept in information theory is entropy, which is a metric for the degree of causality or uncertainty that random events have. Entropy and amount of information are related as follows: the greater the amount of information, the greater the disorder and the greater the entropy; the smaller the amount of information, the smaller the choice and the smaller the entropy.

This talk starts with a brief introduction to Shannon entropy and its main properties. Recall that

$$\mathrm{H}\left(p_1,\ldots,p_n\right)=\sum_{i=1}^{n}p_i\log_2\left(\frac{1}{p_i}\right)=-\sum_{i=1}^{n}p_i\log_2 p_i.$$

is the so-called Shannon entropy for a random event with n possible states, with probabilities p_i, $i=1,...,n$, where $\sum_{i=1}^{n}p_i=1$.

This measure has been applied in different contexts from what was in its genesis. Indeed, entropy is one of several ways to measure biological diversity, and it is applied by biologists and ecologists under the name of Shannon index (*e.g.*, [2] whcih focus an application relating bird species diversity, plant species and foliage heights).

The main purpose of this talk is to highlight an original application of entropy as a tool to measure the diversity of geographic origins of migrating populations [4]. An example of this application to a restricted set of students at the University of Coimbra is presented. Start by grouping students from a given universe (class, subject, course or university) into categories according to their native place of residence. The diversity measure applied in the present context, being a metric for the diversity of geographical origins of the students, will then result from the computation of

$$\mathrm{H}\left(p_1,\ldots,p_N\right)=-\sum_{i=1}^{N}p_i\log_2\ p_i,$$

where p_i stands for the proportion of students belonging to the i-th category and N is the number of geographic categories considered. This example, if extended to a wider universe, may prove to be an important instrument for the characterization of student migration, with practical consequences.

***K*eywords** Biological diversity · information theory · maximum entropy · population migration · Shannon entropy.

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On a New Generalization of Fibonacci Type Hybrid Numbers and Their Properties

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Abstract

In this talk, by using q-integers and higher order generalized Fibonacci numbers, we define the higher order generalized Fibonacci hybrid numbers with q-integer components. Then, we give some algebraic properties of this type of hybrid number.

Keywords Hybrid Numbers · q-integer · Generating function

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ORBIT EXTRAPOLATION

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ABSTRACT

Orbit extrapolation is the calculation of the trajectory of the orbit at time t from knowledge of its initial condition at time t_0. This calculation can be either analytical, ie. be an algebraic formula, a priori result of formal mathematical integration, at time t, or numerical, ie. be the integration of an ordinary differential equation (ODE) step by step from t_0 to t. For many reasons this calculation is not easy to do. Outside of the unperturbed two-body problem, formal integration does not exist. Perturbed problems require high-order numerical integrators; n-body problems, for an orbital trajectory and $n > 2$, also require these efficient numerical integrators. Over the centuries and knowledge of orbital perturbations, the precision of orbit calculation has increased. However, a major challenge remains, that of the duration of the extrapolation. The further the extrapolation is in time, $t >> t_0$, the more the result deteriorates to the point of no longer being good. The 21st century has so far seen a real improvement in orbit extrapolation, even for high and very eccentric orbits. While there is no formal solution to the real orbital trajectory, analytical methods can nevertheless be implemented for so-called average problems or in an approximation framework. Here we summarize some of the most efficient modern analytical and numerical orbit extrapolation methods of the perturbed two-body problem. We will begin by recalling the basics of orbital mechanics, as well as those of the numerical integration of ordinary differential equations. The aim is indeed this review of methods which seems useful to any mechanic in charge of choosing the method of calculating his orbit. This review is also useful to researchers in orbital mechanics by making them aware of methods that are not their own, but also to mathematics teachers. The presentation, although short but synthetic, is the result of years of study and research in the field of space technology. It is a challenge to expose so many techniques in a very short time, but the summary tables will help us.

	PERTURBATIONS	ACCURACY	COMPUTATION TIME	ERROR	EXAMPLE	DEDICATED TO (non-exhaustive)
GENERAL PERTURBATIONS (Analytical integration)	limited number of perturbations	low	very fast computing (at the date)	non-cumulative	SGP4	mission analysis, tracking, catalog
SEMI-ANALYTICAL (Analytical-numerical integration)	all essential perturbations	high	fast computing (big step)	long term cumulative (roundoff)	DSST	long term propagation
SPECIAL PERTURBATIONS (Numerical integration)	all the perturbations	high	long (large number of calculations)	cumulative (truncation, roundoff)	DOPRI853 + forces model	orbit determination

Figure 1: Summary of the three main classes of orbit extrapolation.

Keywords Mechanisc · ODEs · Orbit · Extrapolation · Analytical · Semi-Analytical · Numerical

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RENAL HEALTH AND PATTERNS OF WATER CONSUMPTION: DESCRIPTIVE AND INFERENTIAL ANALYSIS IN CIUDAD HIDALGO

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ABSTRACT

This study examines the relationship between water consumption, dietary habits, and the incidence of renal diseases in Ciudad Hidalgo, Mexico. For the first time, factors such as non-bottled water consumption, dietary practices, and local awareness of renal health are explored and evaluated. In this region, a significant proportion of the population suffers from renal diseases, raising serious public health concerns. This study investigates potential causes, hypothesizing that non-bottled water consumption might contribute to this health issue. To gather relevant data, a survey was designed to assess respondents' beverage consumption, dietary habits, health knowledge, and interest in health awareness.

A 16 Item survey was conducted, yielding 491 responses. Descriptive statistical analysis was conducted to interpret the data. The mean age of respondents was 31 years, with 62% aged between 15 and 30. The survey also revealed that 89% of respondents consume beverages other than water, with 69% reporting infrequent alcohol consumption. However, 78% of respondents reported frequent consumption of junk food, with a notable gender difference: 58% of junk food consumers were female. Additionally, there was a strong demand for health education, with 88% of respondents expressing interest in workshops on dietary habits and health care.

While the survey provided valuable insights into local dietary habits and health awareness, it did not conclusively identify the factors contributing to the high incidence of renal diseases. Some survey questions were ambiguous, leading to varied interpretations and highlighting the need for more precise survey instruments in future research. Despite its limitations, this study lays the groundwork for further research and intervention programs aimed at improving renal health in Ciudad Hidalgo. The findings underscore the need for preventive and educational measures to address this emerging public health issue.

Keywords Statistical analysis · Kidney diseases · Water consumption · Eating habits

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GALOIS BUNDLES AND AUTOMORPHISMS OF THE PRINCIPAL BUNDLE MODULI SPACE

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ABSTRACT

Given a compact Riemann surface X and a semisimple complex Lie group G, Ramanathan [9, 10] constructed the moduli space $M(G)$ of polystable principal G-bundles over X. The study of the geometry and topology of these moduli spaces have several applications in many fields of mathematics, theoretical physics and engineering. First, the holomorphic structures admitted by a given principal G-bundle is in bijective correspondence with the Dolbeault operator space, in the spirit of the Newlander-Nirenberg theorem [11]. Indeed, the latter space is isomorphic to the space of flat connections with values in a maximal compact subgroup H of G, which links principal G-bundles to the solutions of the Yang-Mills equations of quantum field theory [3]. In addition, the moduli space $M(G)$ is isomorphic to the moduli space of representations of the fundamental group of the base curve X in H [8, 7]. Likewise, in technical, applied science and engineering areas, principal bundles are used, among other situations, in contexts of chaotic prediction in engineering problems [6]. One of the most fruitful lines of research concerning the geometry of the moduli space of principal G-bundles is the study of their automorphisms and, in particular, the identification and analysis of their fixed points subvarieties [1]. Fringuelli [4] proved that there are three families of automorphisms of $M(G)$ that generate, by composition, the whole group of automorphisms: the action of an outer automorphism of G; the action, by pull-back, of an automorphism of X; and the action, by tensor product, of line bundles parametrized by $H^1(X, Z)$, where Z denotes the center of G. The literature has extensively studied, above all, automorphisms of the first type, in different contexts: outer involutions in the moduli space of vector bundles [5] or actions of outer automorphisms of G when G is a simple group [1]. In the present work, automorphisms defined as a composition of the action of an outer automorphism of G of order 2 with the action of an involutive automorphism of the base curve X are considered. Thus, it will be assumed that G admits only one nontrivial outer automorphism σ, which has order 2 (this is the case, for example, of the special linear group) and that X is a hyperelliptic curve, so that σ acts as the hyperelliptic involution of X. Specifically, this work provides a description of the fixed points of this type of automorphisms. To do this, the notion of Galois G-bundle, introduced in [2] for the specific case of principal bundles with structure group E_6, is used. The principal G-bundles E that admit a Galois structure are exactly the fixed points of the mentioned automorphism such that the iteration of the isomorphism $E \to \sigma^*(\sigma(E))$ is the identity $E \to E$. Therefore, a generalization to semisimple Lie groups of the results proved in [2] is provided here. Specifically, for a Galois G-bundle that admits a nontrivial automorphism commuting with the Galois structure, it is provided a reduction of its structure group to the centralizer of a non-central semisimple element of G. From this, a vector form of the Galois G-bundles above is induced, given a faithful representation of G. Furthermore, the relationship that exists between the Galois G-bundles and the Galois G/Z-bundles is discussed, Z being the center of G. Specifically, sufficient conditions are provided for a G/Z-bundle Galois structure to lift a G-bundle Galois structure.

Keywords Galois bundle · Principal bundle · Automorphism · Fixed point · Lie group · Semisimple

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ON CONSTRUCTING FRAME FIELDS OF A SPLIT OCTONIONIC CURVE WITH AN ASSOCIATED SPATIAL SPLIT OCTONIONIC CURVE

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ABSTRACT

In this presentation, we will define an associated spatial split octonionic curve for a given split octonionic curve. We construct frame formulas for the split octonionic curve by using the G_2- frame and Frenet-Serret frame of the associated spatial split curve.

***K*eywords** Frenet-Serret Frame · G_2 Frame · Derivative formulas

Funding
This research has been supported by Istanbul Beykent University Scientific Research Projects Coordination Unit. Project Number: 2023- 24-BAP-01, 2024.

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A New Approach to Coding Theory

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Abstract

In this paper, we initially investigate k-Narayana sequence and then define negative indexed k-Narayana sequence. Moreover we obtain some interesting properties for them. Furthermore, we give a new approach to the coding theory with k-Narayana sequence. Finally, we illustrate the coding steps with two examples and verify the results via software.

***K*eywords** k-Narayana sequence · Coding · matrix representation

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LINEAR RECURRENCES AND DECOMPOSABLE FORMS

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ABSTRACT

Let $k \geq 2$ denote an integer. Assume that there are given k distinct, linearly independent, homogeneous linear recurrences of order k satisfying the same recurrence relation. These recurrences are related to a decomposable form of degree k, and there is a general identity with a suitable exponential expression depending on the recurrences. This identity is a common and very broad generalization of several known identities. If the recurrences are integer sequences, then the diophantine equation associated to the decomposable form and the exponential term has infinitely many integer solutions generated by the terms of the recurrences. A method for the complete factorization of the decomposable form can be given. Both the form and its decomposition are found if $k = 2$, and a few examples for $k = 3$ are shown.

An example is provided here with two binary recurrences. Let $G_0 = 1$, $G_1 = 2$, $H_0 = 3$, $H_1 = 4$, and both sequences (G_n) and (H_n) satisfy the recurrence rule $x_n = 4x_{n-1} - x_{n-2}$. Then the identity

$$23G_n^2 - 18G_nH_n + 3H_n^2 = -4 \tag{1}$$

holds, where the left-hand side is the corresponding (quadratic) form depends on the initial values and the coefficients of the recurrences. Equality (1) resembles us the well-known identity

$$L_n^2 - 5F_n^2 = \pm 4,$$

where (F_n) denotes the Fibonacci sequence and (L_n) is its associate or companion sequence (i.e. sequence of Lucas numbers). But in case of (1) we had no prescribed connection (like associate sequence) between (G_n) and (H_n), only the common recurrence rule was fixed. It turned out that there exists a similar identity for any two recurrences with property $x_n = 4x_{n-1} - x_{n-2}$, and this identity can be given explicitly.

Keywords linear recurrence · decomposable form · general identity · diophantine equation.

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INTUITIONISTIC FUZZY WEAKLY 2-ABSORBING IDEALS OF A LATTICE

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ABSTRACT

In this study, we investigate the notion of an intuitionistic fuzzy prime ideal and intuitionistic fuzzy weakly 2-absorbing ideal of a lattice as an expansion of a notion of a weakly prime ideal. Then, we characterize the concept of intuitionistic fuzzy weakly 2-absorbing ideals and intuitionistic fuzzy weakly primary ideals, and give some results of them. Finally, we suggest the concept of an intuitionistic fuzzy weakly 2-absorbing ideals in a product of lattices.

***K*eywords** Lattice · Intuitionistic Fuzzy Ideal · Intuitionistic Fuzzy Weakly Prime Ideal · Intuitionistic Fuzzy 2-Absorbing Ideal

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ON CHEMICAL GRAPHS AND THEIR APPLICATIONS TO CODING THEORY

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ABSTRACT

In this paper, by exploiting matrix representation of chemical graphs, an encryption method is proposed for maintaining security of messages with three levels. The first level of this encryption method benefits from the adjacency and Laplacian matrices of chemical graphs. Then, the message is further encrypted using Jacobsthal numbers (taken in min matrix form) by subtracting from the corresponding elements of the product of the adjacency matrix of the chemical graph and message (taken as circulant matrix). Here the chemical graph can be taken as private key. The matrix so formed can be treated as an adjacency matrix which in turn can be represented as a graph and will be sent to the receiver as super encrypted message. The receiver writes the adjacency matrix corresponding to the graph, then adds the Jacobsthal matrix from adjacency matrix and then multiplies it with the inverse of the obtained matrix C using the shared private keys Finally, receiver decrypts the message using the private key shared.

Keywords Graph · Circulant matrix · Min matrix · Jacobsthal Numbers · Cryptography

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ESCAPE ROOMS AND STUDENTS COMPETENCIES

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ABSTRACT

This paper presents the possible impact of gamification on the acquisition of mathematical competencies in the context of Differential and Integral Calculus. Gamification, the incorporation of game design elements in educational settings, serves as a powerful pedagogical tool to enhance student engagement and motivation. By integrating game mechanics, and challenges into the learning process, this approach aims to create an interactive and stimulating educational environment. The study examines how gamification may facilitate the understanding and application of calculus concepts, promoting critical thinking and mathematical competencies as outlined in Niss's competency framework. In this study, students engage with calculus content through a series of carefully designed game-based activities and assessments. These activities are embedded within a narrative where students are tasked with planning and rebuilding a city devastated by a catastrophe. The primary educational objective is to master integral and differential calculus concepts through this immersive and interactive approach. According to Niss's framework, competencies such as thinking mathematically, problem-solving and posing, modelling, reasoning, use of aids and tools, and communication are expected to be enhanced. The gamified approach also aims to enhance collaborative learning and improve communication among peers, contributing to the development of essential students 'soft skills. This paper proposes gamification as an activity to be integrated into the calculus curriculum since the authors believe that it will not only enhance the acquisition of mathematical knowledge but also support their proficiency in Differential and Integral Calculus. It offers insights into innovative pedagogical approaches for educators.

***K*eywords** Education · Mathematics · Gamification · Competencies · Escape Rooms

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DE MOIVRE'S FORMULA FOR THE MATRICES OF GENERALIZED OCTONIONS (CAYLEY NUMBERS)

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ABSTRACT

This article focuses on the matrices of generalized octonions, also called Cayley numbers, which include real octonions, split octonions, semi octonions, split semi octonions, quasi octonions, split quasi octonions and para octonions in special cases. We give the matrix representations of the generalized octonions with some algebraic properties. Also, we present formulae for powers and roots of the matrix representations.

***K*eywords** Cayley numbers · 3-parameter generalized octonions · De Moivre formula

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USING SOLO TAXONOMY TO DEVELOP A STRUCTURED DESIGN FOR MATHEMATICS EXAM QUESTIONS.

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ABSTRACT

Evaluation is a complex task that requires clear and transparent criteria, ensuring all involved understand it as credible, with educational and social responsibility. However, as evaluation is not an exact science, it is naturally subjective. Do teachers prepare questions covering all levels of complexity? Do they reflect on the knowledge and complexity required for each question? What methods and tools do they use? In higher education, studies on evaluation are few, and in mathematics subjects, they are almost nonexistent. Greater reflection on this topic in higher education institutions is needed to deepen knowledge. The SOLO taxonomy, developed by John Biggs and Kevin Collis [1], in 1982, includes five levels of learning complexity: pre-structural, uni-structural, multi-structural, relational, and abstract. The authors have used this taxonomy to assess exam quality and identify cognitive complexity levels needed for assessments [2, 3]. The SOLO taxonomy should be used by teachers to formulate questions, classifying them according to cognitive complexity and assigning appropriate weights in student evaluations [4]. This approach helps identify areas for student improvement, aiming for greater academic and professional success. In this paper, the design approach adopted for the structured development of questions in a mathematics exam is described, following an ascending process of cognitive complexity. The exam questions are structured to flow in a sequence starting with some items of lower complexity and gradually becoming more challenging. By following this pattern, students engage with questions that vary in levels of progressive cognitive complexity, which also helps them build confidence as they progress towards more complex and challenging tasks.

***K*eywords** SOLO taxonomy · cognitive complexity · assessment · levels of cognitive knowledge · mathematics · higher education

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A NOTE ON ANALYTICAL SOLUTIONS OF ONE OF THE SPECIAL FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION

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ABSTRACT

This work utilizes the modified $G^{'}$-expansion approach to provide analytical solutions for the nonlinear fractional integro-differential Kadomtsev-Petviashvili (KP) hierarchy equation with conformable time-fractional derivative. The modified $G^{'}$-expansion technique is a useful and significant approach for finding analytical solutions to nonlinear equations. With the proposed method, there are three unique types of solutions such as hyperbolic, trigonometric and rational solutions. These solutions have been found with the help of software program.

***K*eywords** KP Hierarchy equation · Conformable derivative · Modified $G^{'}$-expansion method

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ON THE BI-PERIODIC EDOUARD AND THE BI-PERIODIC EDOUARD–LUCAS NUMBERS

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ABSTRACT

Recently, several families of sequences of numbers have been studied, such as Fibonacci, Pell, Lucas, Leonardo numbers, and their generalizations. One of these interesting sequences is the Edouard and Edouard–Lucas numbers, $\{E_n\}_{n\geq 0}$ and $\{E_n\}_{n\geq 0}$, defined by the recurrence relation and initial conditions

$$E_n = 7E_{n-1} - 7E_{n-2} + E_{n-3}, E_0 = 0, E_1 = 1, E_2 = 7, \tag{1}$$

$$K_n = 7K_{n-1} - 7K_{n-2} + K_{n-3}, K_0 = 3, K_1 = 7, K_2 = 35. \tag{2}$$

respectively. Relations (1) and (2) can be rewrote as a non-homogeneous relations, given below

$$E_n = 6E_{n-1} - E_{n-2} + 1, E_0 = 0, E_1 = 1,$$

$$K_n = 6K_{n-1} - K_{n-2} - 4, K_0 = 3, K_1 = 7.$$

The Edouard and Edouard-Lucas numbers was explored in the literature (see more in [4] and references there in).

Various generalizations of known number sequences have also been considered. For example, for any real nonzero numbers a and b, Edson and Yayenie [2] introduced the generalization of the Fibonacci sequence. The bi-periodic Fibonacci numbers $\{F_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$F_0^{(a,b)} = 1,\ F_1^{(a,b)} = 1,\ F_n^{(a,b)} = \begin{cases} 6F_{n-1}^{(a,b)} + F_{n-2}^{(a,b)}, & \text{if } n \text{ is even} \\ 6F_{n-1}^{(a,b)} + F_{n-2}^{(a,b)}, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\,. \tag{3}$$

Other works of literature explore this type of generalization, see [1, 3].

When $a = b = 1$, in Equation (3), we have the classical Fibonacci sequence, and for $a = b = 2$, we get the Pell numbers. If we set $a = b = k$ for some positive integer k, we come to the k-Fibonacci numbers.

In this study, we introduce two new sequences: the bi-periodic Edouard and the bi-periodic Edouard–Lucas numbers.

Definition 1. For any two non-zero real numbers a and b, the bi-periodic Edouard numbers $\{E_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$E_0^{(a,b)} = 0,\ E_1^{(a,b)} = 1,\ E_n^{(a,b)} = \begin{cases} 6aE_{n-1}^{(a,b)} - E_{n-2}^{(a,b)} + a, & \text{if } n \text{ is even} \\ 6bE_{n-1}^{(a,b)} - E_{n-2}^{(a,b)} + b, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\,.$$

The first six elements of the bi-periodic Edouard numbers are

$E_0^{(a,b)}$	$E_1^{(a,b)}$	$E_2^{(a,b)}$	$E_3^{(a,b)}$	$E_4^{(a,b)}$	$E_5^{(a,b)}$
0	1	$7a$	$42ab + b - 1$	$252a^2b + 6ab - 12a$	$1512a^2b^2 + 36ab^2 - 114ab + 1$

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When, $a = b = 1$ we have the classic Edouard numbers.

Definition 2. For any two non-zero real numbers a and b, the bi-periodic Edouard–Lucas numbers $\{K_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$K_0^{(a,b)} = 3,\ K_1^{(a,b)} = 7,\ K_n^{(a,b)} = \begin{cases} 6aK_{n-1}^{(a,b)} - K_{n-2}^{(a,b)} - 4a, & \text{if } n \text{ is even} \\ 6bK_{n-1}^{(a,b)} - K_{n-2}^{(a,b)} - 4b, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\ .$$

The first six elements of the bi-periodic Edouard–Lucas numbers are

$K_0^{(a,b)}$	$K_1^{(a,b)}$	$K_2^{(a,b)}$	$K_3^{(a,b)}$	$K_4^{(a,b)}$	$K_5^{(a,b)}$
3	7	$38a-3$	$228ab-22b-7$	$1368a^2b-132ab-84a+3$	$8208a^2b^2-792ab^2-732a+36b+7$

When, $a = b = 1$ we have the classic Edouard numbers.

In addition, we establish some properties, identities, and recurrence relations of these sequences. The relation with the Balancing and Lucas–Balancing numbers are explored and some identities involving these sequences are provided.

***K*eywords** Balancing sequence · Edouard numbers · Edouard–Lucas numbers

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A BRIEF STUDY ON THE TRIDIMENSIONAL LUCAS-COBALANCING NUMBERS

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ABSTRACT

Currently, several sequences of numbers have been investigated by many researchers. One of these sequences is the Lucas-cobalancing numbers $\{c_n\}_{n\geq 1}$ which are obtained through recurrence relations

$$B_{n+2} = 6B_{n+1} - B_n,$$

with initial values $B_0 = 0$ and $B_1 = 1$, and

$$c_{n+2} = 6c_{n+1} - c_n,$$

with initial values $c_1 = 1$ and $c_2 = 7$, respectively. (See for more information the works [1, 2, 5, 6, 9, 10], among others).
Many other versions of sequences have also been studied, such as the bidimensional versions of balancing $\{B_{(n,m)}\}_{n,m\in\mathbb{N}}$ and Lucas-cobalancing $\{c_{(n,m)}\}_{n,m\in\mathbb{N}}$ which are given by the recurrence relations

$$\begin{cases} B_{(n+1,m)} &= 6B_{(n,m)} - B_{(n-1,m)}, \\ B_{(n,m+1)} &= 6B_{(n,m)} - B_{(n,m-1)}, \end{cases}$$

with initial conditions $B_{(0,0)} = 0$, $B_{(1,0)} = 1$, $B_{(0,1)} = i$, $B_{(1,1)} = 1 + i$ and $i^2 = -1$, and

$$\begin{cases} c_{(n+1,m)} &= 6c_{(n,m)} - c_{(n-1,m)}, \\ c_{(n,m+1)} &= 6c_{(n,m)} - c_{(n,m-1)}, \end{cases}$$

with initial conditions $c_{(0,0)} = 1$, $c_{(1,0)} = 7$, $c_{(0,1)} = i$, $c_{(1,1)} = 7 + i$ and $i^2 = -1$, respectively. (For more information, see, among others, [3, 4]).
In this work, we introduce the tridimensional version of Lucas-cobalancing numbers. We also study some properties and some sum identities.

Keywords Balancing numbers · Lucas-cobalancing numbers · Tridimensional version · Properties

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INNOVATIVE DIGITAL APPROACHES IN MATHEMATICS EDUCATION

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ABSTRACT

This paper explores the transformative impact of digital technologies on mathematics education within the DigiSTEM framework, with a specific focus on higher education. This paper delves into the integration of cutting-edge digital tools, learning analytics, and intelligent assessment methods to enhance pedagogical practices and the digital proficiency of mathematics educators. The study addresses the evolving role of technology in reshaping traditional teaching methods, emphasizing its potential to foster active learning environments, promote self-regulated learning processes, and enhance student engagement. By leveraging digital innovations, such as interactive simulations, virtual laboratories, and adaptive learning platforms, educators can tailor instructional approaches to meet diverse student needs and learning styles effectively. Central to the investigation is the DigiSTEM initiative's role in advancing mathematics education through modern pedagogical strategies. The initiative not only facilitates the adoption of digital tools but also encourages collaborative learning experiences that bridge theoretical knowledge with real-world applications. Through case studies and empirical research, this paper illustrates how these innovations empower educators to create dynamic learning environments that stimulate critical thinking and problem-solving skills among students. The paper discusses the implications of integrating digital technologies in higher education mathematics instruction, including challenges such as access to resources, faculty training, and equitable implementation across educational contexts. It underscores the importance of ongoing professional development for educators to effectively harness the potential of digital tools and maximize their impact on student learning outcomes. By examining successful implementations and emerging trends within the DigiSTEM framework, this paper offers insights into best practices and practical recommendations for stakeholders in mathematics education. It advocates for strategic investments in digital infrastructure and educational support systems to sustain innovation and drive continuous improvement in mathematics pedagogy. This exploration of innovative digital approaches within DigiSTEM underscores their role in shaping the future of mathematics education, paving the way for inclusive, engaging, and effective learning experiences that prepare students for success in a digital era.

***K*eywords** Mathematics Education · Digital Approaches · Pedagogical Strategies · Higher Education

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MODIFIED HUNGARIAN METHOD (MHM) IN OPTIMIZING COMPETENCY-PREFERENCES SCORES IN LECTURER-TO-COURSE ASSIGNMENT

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ABSTRACT

Efficient assignment of lecturers to courses is vital in educational institutions to ensure both faculty satisfaction and optimal course delivery. This paper proposes a Modified Hungarian Method (MHM) optimization model for the assignment of lecturer-to-course considering their competency and preference scores. Previous research predominantly employed the Hungarian method, with a limited exploration of the MHM optimization model. Furthermore, the combination of competency and preference-based lecturer-to-course has never been applied. In the field of assigning lecturers to courses, previous research often used the Hungarian method to solve balanced assignment problems, where the number of lecturers and courses is equal. However, studies addressing unbalanced assignments regarding assigning lecturers to courses are still lacking. This study focuses on tackling this unbalanced assignment issue, providing solutions for situations where the numbers do not align and solving it based on the competency and preferences of the lecturers. To enhance the formulation of the MHM model, this study presents a mathematical programming approach. The objective of this study is to maximize overall competency and preferences CP-MHM model in lecturer-to-course assignments. Competency and preferences data from Mathematics lecturers at UiTM Shah Alam were gathered via an online survey for undergraduate courses. Using these competency and preference scores as input, the CP-MHM model was implemented using MATLAB's intlinprog to produce an optimal assignment plan, restricting lecturers to a maximum of three courses. The optimal solutions of the CP-MHM model indicate which courses are best assigned to a particular lecturer, based on their competency and preference scores. The competency levels are assessed using three aspects namely knowledge, skills, and teaching motivation. This research improves educational planning by providing a useful tool for assigning lecturers to courses in realworld situations. A key part of our study is combining the assignment process with the MHM algorithm to handle these multidimensional inputs effectively. By considering both lecturer preferences and competencies, as well as the available courses, our model aims to boost teaching quality, reduce mismatches, and enhance overall academic performance.

***K*eywords** Competency · Preferences · Higher Education Institutions · Lecturers-to-Courses · Assignment · Mathematical Programming · Modified Hungarian Method

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ON THE (r, p, k)-GENERALIZED JACOBSTHAL NUMBERS AND THE JACOBSTHAL FUNDAMENTAL FIBONACCI SYSTEM

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ABSTRACT

Recently, several research papers has been produced on the classic sequences of additive theory such as the Fibonacci-Lucas numbers, the Pell numbers and the Jacobsthal numbers. Although they are theoretical in nature, these research papers were motivated by the numerous applications of these sequences of numbers and their various generalizations, in different mathematical and applied mathematics fields, as well as in the exact and applied sciences, and in art. (see, for example, [2, 3] and references therein). This talk deals with a model of (r, p, k)-generalized Jacobsthal numbers by the approach based on the properties of its associated Jacobsthal fundamental Fibonacci system. Some linear and combinatorial properties are established. Moreover, the related matrix formulation allows us to provide new identities. Especially, the linear Jacobsthal Cassini identity and its combinatorial formulation are furnished. Finally, illustrative special cases and examples are given. More precisely, the model of (r, p, k)-generalized Jacobsthal numbers is defined by

$$\begin{cases} J_{n+1} = J_n + 2^p J_{n-1} + J_{n-2} + \cdots + J_{n-r} + 2^k J_{n-r+1}, \text{ for } n \geq r, \\ J_0 = \alpha_0, \cdots, J_{r-1} = \alpha_{r-1}, \end{cases} \tag{1}$$

where $\alpha_0, \cdots, \alpha_{r-1}$ are the initial conditions and k, p are given integers in $\mathbb{N}$. We can observe that for $p = 1$ and $k = 0$ or $p = 0$, $k = 1$ in (1), we get known following generalized Jacobsthal numbers

$$J_{n+1} = J_n + 2J_{n-1} + J_{n-2} + \cdots + J_{n-r} + J_{n-r+1}, \text{ for } n \geq r, \tag{2}$$

$$J_{n+1} = J_n + J_{n-1} + J_{n-2} + \cdots + J_{n-r} + 2J_{n-r+1}, \text{ for } n \geq r, , \tag{3}$$

where $J_0 = \alpha_0, \cdots, J_{r-1} = \alpha_{r-1}$ are the arbitrary initial conditions. In addition, for $p = k = 0$ Expression (1) is reduced to the well known generalized Fibonacci numbers of order r studied in various research papers (see, for example, [1, 4], and references therein).

***K*eywords** Generalized Jacobsthal numbers · (r, p, k)-Generalized Jacobsthal numbers · Jacobsthal Fundamental Fibonacci System · matrix representation · combinatorial formulas · Jacobsthal Cassini identity.

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THE ALPHA STAR-PERFECT MAPPING IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce new types of $\alpha^\star$-continuity mappings by using $\alpha^\star$-open sets in topological spaces which is called alpha*-perfect mapping, also we study some properties of these types. Some definitions are given. We show that if $f : X \to Y$ is $(\alpha^\star\text{-}PM)$. Then the restriction of f on clopen set is also $(\alpha^\star\text{-}PM)$. Next, for any mapping $f : X \to Y$ between two topological spaces with A, B are disjoint clopen sets, where $X = A \cup B$. In this work, we show that $f|_A$ and $f|_B$ are $(\alpha^\star\text{-}PM)$ if and only if f is $(\alpha^\star\text{-}PM)$. Finally, we prove that if $f : X \to Y$ and $g : U \to W$ are mappings with $f \times g : X \times U \to Y \times W$ is $(\alpha^\star\text{-}PM)$, then each one of f and g is $(\alpha^\star\text{-}PM)$.

***K*eywords** $alpha$-open sets, $\alpha^\star$-open sets, perfect mapping, continuity mapping, $\alpha^\star$-irresolute continuous mapping.

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COMPLEX POTENTIALS SOLUTIONS FOR ISOTROPIC COSSERAT BODIES WITH VOIDS

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ABSTRACT

The aim of this paper is to obtain solutions in terms of complex potentials structure for the plain strain problem, of an elastic micropolar, homogeneous and isotropic body with pores within the equilibrium theory. Using the constitutive equations geometric equations, and the equilibrium equations without body forces, the focus is on addressing the fundamental boundary value problems of plane strain theory. Subsequently, a depiction of the displacement of microrotations and pores is derived using complex analytical functions and two real functions, based on the homogeneous Helmholtz equations. In the next part of the work, the structure of the potential functions for several domains of interest is studied, and the method of complex variables without the introduction of stress functions is applied to solve the Kirch problem. The last section is dedicated to the numerical study, where the corresponding complex potential plots and stress and displacement distributions are obtained in a porous micropolar isotropic material.

***K*eywords** Complex · Potentials · Isotrop · Elasticity · Strain · Micropolar

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ON CIRCULANT MATRICES WITH FIBONACCI QUATERNIONS

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ABSTRACT

In literature, there exists many papers which compute determinant and some kind of norms of circulant matrices involving some well-known number sequences. At this paper, we obtain an explicit formula for circulant matrix involving the well-known Fibonacci quaternions. Then, we compute determinants of these matrices by exploiting the set of orthogonal polynomial, Chebyshev polynomials of the second kind. Moreover, we obtain Euclidean and Spectral norms of these matrices.

***K*eywords** Circulant matrices · Fibonacci quaternions · Chebyshev polynomials · Euclid norms · Spectral norms

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STUDENTS WHO PROGRESS TO HIGHER EDUCATION STEM DEGREES FROM THE IRISH FURTHER EDUCATION SECTOR

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ABSTRACT

In this research we analyse Irish Further Education graduates who progress to Higher Education STEM degrees. In particular we will focus on their level of mathematical preparedness for such a progression. An overview of the Irish Further Education system is presented and comparison is made with several over European systems. We present an analysis of the Irish Further Education sector, how mathematics is taught and assessed in the sector, and existing progression pathways from this sector into HE STEM degrees. Data has been collected from Further Education students aiming to progress to Higher Education, in addition to data from students aiming to make similar progressions from the secondary school sector. A comparison is made between the academic performance of FE graduates relative to other entrants to HE STEM degrees. A student survey consisting of three main parts has been devised. The first part collects demographic information for profile-building, the second part will contain the Indiana Belief Scales Instrument (Stage & Kloosterman, 1992), a questionnaire which has been used in Mathematics Education research internationally to assess students' 'beliefs' around mathematics, and finally the survey will contain a short diagnostic test of key mathematical skills, designed for the specific purpose of this research. Full details of the student survey will be presented along with our initial findings.

***K*eywords** Maths Education · Pathways · Maths Education

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WAVE PROPAGATION WITH TWO DELAY TIMES IN AN ISOTROPIC POROUS MICROPOLAR THERMOELASTIC MATERIAL

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ABSTRACT

In this paper, we are folowing the plane time-harmonic waves propagation in an entire linear thermoelastic space, knowing the wavelength. Concerning the thermodynamic response, we fit the dual phase-lag model, while the effect of porosity on elasticity is given by Cowin-Nunziano theory.

We obtain two shear waves and five longitudinal waves as: quasi-elastic wave, quasi-microrotational wave quasi-micropolar wave, quasi thermal mode, quasi-phase-lag thermal mode. The purpose of numerical simulations and of graphs is to identify the influence of connection between thermoelasticity, microrotation and porousity. the physical features of thermomechanical bodies are investigated in short time intervals.

With the help of the Mathematica software, we managed to obtain some graphs, in the cases: uncoupled, poroelastic with microrotations, thermoelastic with microrotations and full coupled with microrotations, which highlight the changes of the mentioned waves following the couplings. In the last mentioned case it was observed that all waves are subject to damping in time.

The emphasis is on the transmission of thermal energy, the evolution from a single relaxation time to dual phase-lag model (DPL) being obviously necessary. So, τ_q and τ_θ are introduced as two delay times of the analyzed model, more precisely they represent characteristics that involve the achievement of thermal equilibrium, as well as the existence of collisions between electrons and photons.

***K*eywords** dual phase-lag model · wave · isotropic · porous · micropolar · thermoelastic

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AN OPTICAL SOLITON SOLUTIONS OF THE TIME FRACTIONAL PERTURBED GERDJIKOV-IVANOV EQUATION

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ABSTRACT

The fractional perturbed Gerdjikov-Ivanov equation has an important place in examining the effects of optical pulse propagation. It is investigated in this study.The method called (G') extension is used for solving conformable time fractional derivative Gerdjikov-Ivanov equation.With the proposed method, there is an optical soliton solutions that are important place in mathematical physics. These solutions have been found and graphical representation is provided for some special values by using mathematical programming.

***K*eywords** Gerdjikov-Ivanov equation · conformable derivative · G' extension method

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ON THE HYBRID NUMBERS AND GENERALIZED FIBONACCI SEQUENCES

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ABSTRACT

In the present talk, we present a study on the r Generalized Fibonacci hybrid sequences. More precisely, we provide the analytic Binet expression for these sequences. New results and sometimes extensions of some results existing in the literature are established. Furthermore, we highlight the special case $r = 2$, and we offer illustrative numerical examples.

***K*eywords** Generalized Fibonacci sequence · hybrid numbers · Analytic formula

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A NUMERICAL APPROACH FOR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION WITH DELAY AND BOUNDARY LAYER

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ABSTRACT

In this study, we investigate the singularly perturbed convection diffusion problem for a class of Volterra integro-differential equation with delay. Differential equations including a small parameter in the highest order term are known as singularly perturbed differential equations. In general, the solutions of such equations exhibit multiscale phenomena. Within certain thin subregions of the domain, the scale of some derivatives is significantly larger than other derivatives. These thin regions of rapid change are called, boundary or interior layers, as appropriate. Such type of equations occur frequently in mathematical problems in the sciences and engineering for example, in fluid flow at high Reynold number, electrical networks, chemical reactions, control theory, the equations governing flow in porous media, the drift-diffusion equations of semi-conductor device physics, and other physical models [R4, R6]. On the other hand, Volterra delay integro-differential equations arise widely in mathematical models of biology, medicine, physics phenomena. In particular, the propagation of nervous impulse, population dynamics, polymeric liquids can be modelled by these equations [R3,R5].

Typically, the solutions of such problems include boundary or initial layer(s). The classical discretization methods to solve singularly perturbed problems do not work well and fail to give analytical solution when tends to zero. For this reason, it is necessary to develop suitable numerical methods that are uniformly convergent with respect to to solve this type of problems [R2,R4]. There are many studies in the literature using different numerical methods to solve first order delay Volterra integro-differential equations [R1]. However, we did not find any work for the second order delay Volterra integro-differential equations. This gap constitutes the motivation of this study.

To numerical approach for the problem, we construct a fitted difference scheme on a piecewise uniform mesh and analyze the error estimates. We prove that the method is almost first-order convergent with respect to the perturbation parameter in the discrete maximum norm. Moreover, we present the numerical experiments which support the theoretical results.

***K*eywords** Volterra delay integro-differential equation · Boundary layer · Finite difference method · Uniform convergence

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MACHINE LEARNING APPROACH FOR PREDICTING UNDER-FIVE MORTALITY DETERMINANTS IN SOMALILAND

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ABSTRACT

Child mortality is a critical indicator of socioeconomic and demographic development, particularly in regions such as Somaliland in sub-Saharan Africa. This study aims to identify the determinants of under five mortality in Somaliland using machine learning techniques. Data from the 2020 Somaliland Demographic and Health Survey (SDHS) is analyzed, encompassing a comprehensive set of variables at the individual, household, and community levels. Machine learning algorithms are employed to identify key risk factors. The study reveals significant determinants of under-five child mortality in Somaliland. Male children have a higher mortality rate (58%) compared to females (42%), as shown in Figure 1. Twins face a higher mortality risk (63%) compared to non-twins (37%), as illustrated in Figure 2. Higher household wealth is associated with lower mortality rates, with the wealthiest households experiencing a mortality rate of 26% compared to 46% in the poorest households (Figure 3). Children of mothers in nomadic areas face a higher mortality risk (55%) than those in settled areas (45%). This study emphasizes the influence of socioeconomic and demographic factors on under-five child mortality in Somaliland, shedding light on important determinants. Notably, there is no significant link found between maternal education and under-five mortality. However, factors such as wealth index, region of residence, birth order, and twinning are identified as significant determinants. These findings provide valuable insights for policymakers and stakeholders in their efforts to reduce child mortality rates in Somaliland. This study identifies key determinants of under-five child mortality in Somaliland, emphasizing the need to address socioeconomic disparities. Targeted interventions focusing on factors such as twin births, household wealth, birth order, and geographic disparities can significantly reduce child mortality rates in the region.

Keywords Under-five mortality · Determinants · Somaliland · Machine learning · Socioeconomic factors · Sex of the child.

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ANALYSIS OF PROGNOSTIC FACTORS IN PROSTATE CANCER - A NEW APPROACH.

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ABSTRACT

Prostate cancer stands as one of the most prevalent malignancies diagnosed in men, second only to lung cancer in terms of mortality. It represents a significant health concern, particularly given its high prevalence among older men. Despite advances in diagnostic and therapeutic strategies, the accurate prognostication of prostate cancer outcomes remains a critical challenge in clinical practice. Prognostic evaluations often hinge on a complex interplay of biological, clinical, and pathological parameters, necessitating robust tools to guide clinical decision-making.

In our previous work, we undertook a comprehensive study of 300 patients who underwent retropubic radical prostatectomy, drawing data from the "Prof. Dr. Th. Burghele" Clinical Hospital and "St. Ioan" Hospital between January 2008 and December 2009. Our primary analytical tool was logistic regression, through which we aimed to identify key prognostic factors. Notably, we found that an accurate Gleason score was paramount in the pathology report, serving as a critical determinant of patient prognosis. However, despite these findings, the questionnaire we applied during this study did not yield conclusive results, indicating a need for refinement and further development.

Building upon this foundational work, our current study aims to enhance the prognostic evaluation process by developing and testing a new, more effective questionnaire. This effort is driven by the recognition that existing tools, while valuable, have significant limitations. These limitations include insufficient sensitivity to certain prognostic factors and a lack of comprehensive integration of clinical and biological parameters.

The development of our new questionnaire involved several critical steps. Initially, we conducted an extensive literature review to identify gaps in current prognostic tools and to gather insights into potential areas for improvement. This review highlighted the need for a more nuanced approach that could better capture the complex interactions between various prognostic factors.

Our revised questionnaire was designed to address these gaps. It incorporates a broader range of variables, including detailed clinical histories, advanced imaging findings, and molecular markers. The inclusion of these parameters was based on their established relevance in prostate cancer prognosis, as evidenced by recent research. Additionally, we aimed to create a tool that was both comprehensive and user-friendly, ensuring that it could be seamlessly integrated into clinical practice without imposing undue burdens on healthcare providers or patients.

To validate our new questionnaire, we again drew upon the cohort from the "Prof. Dr. Th. Burghele" Clinical Hospital and "St. Ioan" Hospital. This allowed us to maintain consistency with our previous study while also benefiting from a well-documented patient population. We employed logistic regression as our primary analytical method, given its robustness in handling binary outcomes and its widespread use in medical research.

The preliminary results from our new questionnaire were promising. We found that it significantly improved the predictive accuracy of our logistic regression models. Specifically, the inclusion of additional clinical and molecular markers enhanced our ability to stratify patients based on their risk of adverse outcomes. This, in turn, has important implications for patient management, as it allows for more personalized treatment plans that are tailored to individual risk profiles.

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One of the key findings of our study was the reaffirmation of the Gleason score's importance. Despite the addition of new variables, the Gleason score remained a critical predictor of prognosis. This underscores the enduring relevance of traditional pathological assessments while highlighting the value of integrating these assessments with newer, complementary tools.
In addition to the Gleason score, several other variables emerged as significant predictors in our models. These included prostate-specific antigen (PSA) levels, lymph node involvement, and certain genetic markers. The identification of these variables aligns with the broader literature on prostate cancer prognosis and reinforces the importance of a multifaceted approach to prognostic evaluation.
The implications of our findings are far-reaching. By enhancing the accuracy of prognostic assessments, our new questionnaire can contribute to better clinical outcomes. For instance, patients identified as high-risk can be more aggressively monitored and treated, potentially improving survival rates. Conversely, those classified as low-risk may be spared from unnecessary interventions, thereby reducing the incidence of treatment-related complications and improving quality of life.
Moreover, our study highlights the potential for further advancements in this field. As new biomarkers and diagnostic technologies emerge, there is scope for continuous refinement of prognostic tools. Our work sets the stage for future research that can build on our findings, incorporating new insights and innovations to further enhance the accuracy and utility of prostate cancer prognostic assessments.
In conclusion, our study represents a significant step forward in the quest for better prognostic tools in prostate cancer. By addressing the limitations of existing questionnaires and incorporating a broader range of prognostic factors, we have developed a tool that offers improved predictive accuracy. This, in turn, has the potential to enhance patient outcomes through more personalized and effective treatment strategies. Our findings reaffirm the importance of traditional prognostic markers like the Gleason score while also highlighting the value of integrating these markers with newer clinical and molecular data. As we look to the future, we are optimistic that continued research and innovation will yield even greater advancements in the prognostic evaluation of prostate cancer, ultimately contributing to better patient care and outcomes.

***K*eywords** Prostate Cancer · Prognostic Factors · Gleason Score · Logistic Regression · Biostatistics · Predictive Models

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A NOTE ON THE TIME-OPTIMAL CONTROL OF LINEAR SYSTEMS

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ABSTRACT

The time-optimal control of linear systems is one of the problems in the mathematical theory of optimal control and has attracted much attention over the years. The time-optimal control problem aims to find the minimum time in which a controlled object, the movement of which is described by a system of ordinary differential equations can be transferred from a given initial position to a given final p osition. These problems can be thought of as a particular instance of the Bolza or Mayer problems in variational calculus, and they are obtained from these problems by the peculiar form of the functional to be optimized. In this study, we examine the problem of controlling a linear system in a time-optimal manner. We solve a range of these control problems by using the Pontryagin maximum principle which is indeed a kind of first-order necessary conditions on time optimal controls.

***K*eywords** Time-optimal control · Pontryagin Maximum Principle

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ACYCLIC MATRICES WITH A P-SET WITH MAXIMUM SIZE

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ABSTRACT

Given an $n \times n$ symmetric matrix A, $A(\alpha)$ denotes the principal submatrix of A obtained after deleting the rows and columns indexed by $\alpha \subset \{1, \ldots, n\}$. By $m_A(0)$ we mean the nullity of A. If $m_{A(\alpha)}(0) = m_A(0) + |\alpha|$, then we call α a *P-set* of A. In this talk we provide several new characterizations of trees the maximal size of a P-set. We will review the main recent developments and give some illustrative examples.

***K*eywords** Parter vertices · P-set · eigenvalues · multiplicities · nullity · matrices

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BRIDGING ACADEMIC LEARNING AND COMMUNITY SERVICE

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ABSTRACT

This paper outlines a Service-Learning (SL) initiative offered to second-year higher education Mechanical Engineering students to foster mathematical competences and critical soft skills. Service-Learning provides a dynamic framework embracing student development by fusing academic education with real work community needs. By applying mathematical concepts to real-world situations, SL in mathematics education helps students gain a wider appreciation of the practical importance of mathematics and strengthens their critical thinking abilities as they tackle real-world problems and create mathematical solutions or reality interpretations to meet the community's needs. As students interact with their peers and the community, they not only transfer mathematical knowledge but also collaborate on solutions, thereby enhancing their collaboration, communication, and creative skills. Moreover, SL fosters a range of humanistic competencies, such as empathy, leadership, active listening, and community engagement. Through these interactions, students develop a deeper understanding of societal issues and the role of mathematics in addressing them. They learn to appreciate diverse perspectives, leading to a reinforced sense of empathy and cultural awareness. Leadership skills are practiced as students take initiative and assume project responsibility. At the same time, active listening and community involvement are cultivated through direct engagement with community members, ensuring that solutions are relevant and impactful (sometimes in their own lives). Service-learning also promotes self-awareness and personal growth, as students reflect on their experiences and the broader implications of their work. They gain insights into how mathematics can drive positive social change and the importance of their civic engagement. This study describes the integration of SL concepts into the curriculum, with a particular focus on the Statistical Methods course taken by second-year Mechanical Engineering students. The objectives were to develop students' abilities in data analysis, encourage significant community involvement, and evaluate the effect on both their mathematical and humanistic competencies. By bridging academic learning and community service, this initiative aimed to contribute to a full person students'formation who are not only proficient in Statistical Methods but are also equipped with the soft skills necessary for effective and empathetic leadership in their professional and personal lives.

***K*eywords** Education · Mathematics · Service-Learning

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ADVANCED TEACHING STRATEGIES FOR ENHANCING STEM EDUCATION IN HIGHER INSTITUTIONS

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ABSTRACT

This paper explores innovative pedagogical strategies for enhancing STEM education in higher institutions, aiming to equip educators with effective methods to improve student engagement and learning outcomes. The study examines best practices among STEM teachers, highlighting successful strategies such as problem-based learning, flipped classrooms, and collaborative projects. It delves into active learning techniques, emphasizing the EduScrum framework, which adapts Scrum methodology for educational purposes, fostering collaboration and iterative progress. The paper also discusses self-regulation in students, providing strategies for developing essential skills like goal-setting and time management.

Furthermore, the paper reviews innovative assessment methods, contrasting traditional approaches with modern, formative techniques, including project-based assessments and digital portfolios. Emerging trends in teaching engineering mathematics are explored, focusing on integrating real-world applications and leveraging technological advancements. The use of mathematical software in teaching is also examined, showcasing tools like MATLAB and Python for visualization and problem-solving.

By synthesizing these diverse approaches, the paper offers comprehensive recommendations for educators to implement these strategies effectively. The findings aim to contribute to the ongoing development of pedagogical competences in STEM education, ultimately enhancing the quality of teaching and learning in higher education institutions.

***K*eywords** Mathematics Education · Digital Approaches · Pedagogical Strategies · Higher Education

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LINEAR ALGEBRA IN CRYSTAL GEOMETRY, AND VICE VERSA

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ABSTRACT

Linear algebra nowadays belongs to standard undergraduate curricula, both in mathematics studies, and in studies of various applied fields (physics, chemistry, computer science, electrical engineering, civil engineering, ...). This includes classical, more geometric, linear algebra (algebraic operations with Euclidean vectors, analytical geometry of plane and space), as well as abstract linear algebra (abstract vector spaces, matrices, linear maps, ...), both having significant applications and rising importance in modern developments of sciences and technology [11]. This obviously induced research in issues relating to teaching linear algebra [4, 7, 9, 10]. In our presentation we aim to contribute to these studies from a rarely considered perspective, and including observations from more than ten years of teaching courses that include linear algebra topics to chemistry and geology students, where predominantly examples from crystallography are used as a motivation for introducing the techniques, as a realistic application of linear algebra, and as a means to develop the intuition and understanding of abstract concepts.

More precisely, we shall see how analytic geometry with respect to skew coordinate systems and the notion of dual basis appears not only as an application of mathematics to crystallography, but also vice versa. Skew coordinate systems are usually only mentioned in linear algebra courses, but rarely explored, however they have an important and natural role in crystallography. Crystals appear as more or less symmetric polyhedra to the naked eye, but their outer symmetry is the result of their regular (periodic) inner structures, first conjectured by R.-J. Haüy (1743–1822), and confirmed by diffraction experiments in the beginning of the 20th century [3, 8]. The periodicity of crystal structure means that for every crystal we can choose a parallelepiped, called unit cell, such that the whole crystal structure consists of its copies translated for integer multiples of the three vectors spanning it. The three vectors spanning the unit cell of a crystal structure are known as the direct (crystallographic) basis, and in general it is *not* orthonormal. Correspondingly, the crystallographic coordinate system is in general not a Cartesian, but a skew one. This coordinate system is used, among other things, to describe directions of lattice planes, planes passing through three non-colinear points with integer coordinates. Crystal diffraction, the fundamental experimental technique for observing the crystal structure, behaves *as if* it were reflection on lattice planes, and also, due to the laws of crystal growth, crystal faces develop along directions of lattice planes [2, 6, 8].

In the period 1912–1937 P.P. Ewald (1888–1985) showed that the results of crystal diffraction can be modelled in a structure known as reciprocal space, devised earlier (1881) by J.W. Gibbs (1839–1903) [5, 8]. In about the same time the mathematical discipline of functional analysis was founded and among other basic functional analysis concepts the notion of dual space was developed [1]. Gibbs' construction of the reciprocal basis becomes a special case of the dual basis of an inner-product space in this context, thus presenting a realistic, applicable and visualisable application of the abstract dual basis.

While some calculations are well-known for the case of Cartesian coordinates and are standard in linear algebra curricula, it is not always obvious how to carry them out in skew coordinate systems without performing coordinate transformations. Such is the case for calculations of coordinates of normal vectors to planes and of cross-products of two vectors. In this talk we shall describe how the crystallographic reciprocal space construct leads to a simple generalisation of these calculations to general skew coordinate systems (i.e., we apply crystallography to mathematics), and also

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leads, purely mathematically, to the so-called fundamental law of the reciprocal space, which is used e.g. for determining a fundamental crystallographic parameter called the interplanar distance (i.e., we apply mathematics to crystallography). We shall also include some historical remarks as well as examples from teaching practice.

***K*eywords** vector algebra · analytical geometry of space · teaching linear algebra · mathematical crystallography · history of linear algebra · reciprocal space · dual basis · lattice planes

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SERVICE-LEARNING FOR SUSTAINABLE ENERGY ACCESS. INTEGRATING ACTIVE LEARNING AND SDGS IN HIGHER EDUCATION, GIRLS PROJECT.

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ABSTRACT

Service-learning has emerged as a transformative educational approach that integrates community service with academic instruction, emphasizing reflection and critical thinking to enrich the learning experience, teach civic responsibility, and strengthen communities. This paper presents an in-depth examination of service-learning, with a specific focus on its implementation in higher education to address sustainable energy access. By leveraging the case study of the GIRLS project—Generation for Innovation, Resilience, Leadership, and Sustainability—this paper illustrates how service-learning can empower students with the knowledge and skills necessary to tackle real-world challenges. The GIRLS project, developed in collaboration with the Technical University of Civil Engineering Bucharest (UTCB), aims to promote inclusion, diversity, equality, digital transformation, and the Sustainable Development Goals (SDGs) through an innovative gamified structure. The project's primary task is to empower high school and university students with practical knowledge and skills to ensure access to affordable, reliable, sustainable, and modern energy. Central to this initiative is the construction of a prototype energy-independent solar house, which serves as a tangible example of renewable energy application. The solar house project exemplifies the core principles of service-learning by involving students in hands-on activities that directly benefit the community while reinforcing their academic learning. Workshops on energy efficiency, conservation techniques, and renewable technologies are conducted, providing students with prac-tical insights into sustainable energy practices. These activities are designed to be reflective and integrative, allowing students to connect theoretical concepts with real-world applications. Com-munity engagement is another critical component of the GIRLS project. UTCB organizes aware-ness campaigns and public showcase events to educate the broader community about the benefits of sustainable energy. These events create a platform for students to share their knowledge and experiences, thereby fostering a sense of civic responsibility and community involvement. Addi-tionally, the project emphasizes the professional development of educators by introducing them to active learning methodologies, such as research-based learning (RBL), game-based learning (GBL), competency-based learning (CBL), and service-learning (SL). Training sessions equip teachers with the skills to implement these methodologies, making learning more interactive and impactful. The inclusion of Vasco de Quiroga University of Morelia in Mexico adds a unique cross-cultural di-mension to the project. This partnership brings valuable insights from their extensive experience in community service and social innovation, enriching the service-learning experience for UTCB stu-dents and faculty. In conclusion, this paper highlights the significant impact of service-learning in higher education, particularly in the context of sustainable energy access. By integrating academic learning with community service, the GIRLS project not only enhances students' educational expe-riences but also contributes to sustainable development and social cohesion. This approach prepares students to become proactive, responsible citizens capable of driving positive change in their com-munities and beyond.

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***K*eywords** Service-learning · Sustainable energy access · Sustainable Development Goals (SDGs) · Active learning methodologies

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ON DUAL BIQUATERNIONIC SEQUENCE INVOLVING VIETORIS' NUMBERS

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ABSTRACT

A sequence of rational numbers denoted by *Vietoris sequence*, appeard in a result published in [11], presented by L. Vietoris in 1958. The first Vietoris' numbers of the sequence are

$$1,\ \frac{1}{2},\ \frac{1}{2},\ \frac{3}{8},\ \frac{3}{8},\ \frac{5}{16},\ \frac{5}{16},\ \frac{35}{128},\ \frac{35}{128},\ \frac{63}{256},\ \frac{63}{256},\ \frac{231}{1024},\ \frac{231}{1024},\ \cdots,$$

which is related with the sequence A283208 in the OEIS in [9]. This sequence, which will be denoted by $\{v_n\}_{n\geqslant 0}$, is defined as

$$v_n = \frac{1}{2^n}\binom{n}{\lfloor \frac{n}{2} \rfloor}, \qquad n \geqslant 0,$$

and satisfies the following properties:

$$v_n = \begin{cases} 1, & n = 0 \\ \dfrac{n}{n+1}\, v_{n-1}, & n \text{ odd} \\ v_{n-1}, & n \text{ even} \end{cases}.$$

Many properties of Vietoris sequence can be seen in P. Catarino and R. de Almeida in [4] and I. Cação et al. in [2]. Special types of matrices that generates this number sequence were presented also in [4]. This sequence finds applications in harmonic analysis, demonstrated by the work of R. Askey and J. Steinig in [1]. It also plays a role in the theory of stable holomorphic functions, as presented in the paper by St. Ruscheweyh and L. Salinas [10].
In [3], we study some properties of the quaternionic sequence, $\{Q_n\}_{n\geqslant 0}$, with Vietoris' numbers as its components

$$Q_n = v_n + v_{n+1}\mathbf{i} + v_{n+2}\mathbf{j} + v_{n+3}\mathbf{k}, \quad n \in \mathbb{N}, \tag{1}$$

where $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is the standard basis in $\mathbb{R}^4$ satisfying the following multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ij} = -\mathbf{ji} = \mathbf{k}, \quad \mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \quad \mathbf{ki} = -\mathbf{ik} = \mathbf{j}.$$

To construct a dual biquaternionic sequence involving Vietoris's numbers, we start by considering the two-dimensional system of dual numbers defined by

$$\mathbf{D} = \{x + Iy : x, y \in \mathbb{R}, \quad I^2 = 0\}.$$

The conjugate of a dual number $z = x + Iy \in \mathbf{D}$ is defined by $\overline{z} = x - Iy$, and the modulus is defined as $|z| = \sqrt{z\overline{z}} = \sqrt{|x^2|} = |x|$. In [5] the basic properties of dual quaternions including fundamental operations, conjugates, inner product, vector product, and norm are discussed. Dual quaternions, are an extension of dual numbers, where the coefficients of real quaternions are dual numbers. It is well known several applications of dual quaternions, for instance, in [5] is mentioned the applications not only in animation, robotics, computer vision applications, theoretical kinematics, but also in the

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express and analyse of the physical properties of rigid bodies (see also the works [6, 7, 8] for more details). In this paper, using (1), we introduce an dual biquaternionic sequence, $\{V_n^p\}_{n\geqslant 0}$, given by

$$V_n^p = Q_n + IQ_{n+1}, \quad I^2 = 0, \quad n \in \mathbb{N},$$

and discusse some of its properties.

***K*eywords** Biquaternion · Dual numbers · Vietoris' number

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A NEW PERSPECTIVE TO OVERCOME THE CHALLENGE OF SEMI-SUPERVISED CLUSTERING PROBLEM IN MACHINE LEARNING

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ABSTRACT

Machine learning (ML), a subfield of artificial intelligence (AI), can be defined as "learning" methods. In other words, they are methods developed without being explicitly programmed to make predictions and decisions using available data. The study of mathematical optimization offers methods, theory, and application areas for machine learning. For instance, speech recognition [10, 12, 8], internet security [7, 5], road detection [3, 1], image classification [2, 11, 6], information retrieval [4], bioinformatics [9], and many more. These are examples of machine learning for supervised, unsupervised, semisupervised, self-supervised, and reinforcement learning. These are examples of machine learning for supervised, unsupervised, semisupervised, self-supervised, and reinforcement learning. This study focuses on semisupervised learning problems as in [10, 12, 7, 3, 2, 4, 9], in particularly semisupervised clustering. When must-link and cannot-link constraints are given for a clustering problem, this clustering problem is considered a semi-supervised learning problem. Must-link constraints force certain data points to be in the same cluster, while cannot-link constraints prevent some data points from being in the same cluster, which can create complex requirements for the clustering algorithm to balance. Ensuring that these constraints are satisfied while forming coherent and meaningful clusters can be difficult, especially when the constraints are sparse or conflict with the natural structure of the data. Especially when the number of clusters increases, the rate of providing must-link constraints decreases dramatically. Another difficulty is the computational complexity involved in integrating these constraints into the clustering process, particularly for large datasets. The algorithm must efficiently search for clusters that satisfy the given constraints while maintaining overall cluster quality. Must-link constraints form an equivalence relation. In this context, it is possible to separate must-link constraints into equivalence classes. In this presentation, a suggestion based on this equivalence class will be given to satisfy more must-links for the new algorithms.

***K*eywords** Machine Learning · Semi-supervised Clustering · Non-smooth Optimization · Semi-supervised Learning

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ON NEURAL NETWORK ANALYSIS OF DISCRETE FRACTIONAL ORDER CANCER MODEL BY ARTIFICIAL INTELLIGENCE

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ABSTRACT

This article presents the numerical approximate solutions of the novel constructed Difference Caputo Order Cancer (DCOC) model by employing the numerical Euler method combined with the neural networks approach. The DCOC model is systemized for five variables, further classified into four stages to investigate. Various values in fractional order are considered to study the different scenarios of the DCOC model. The iterative systems of the DCOC model have been formulated by the Euler method with neural networks (ENNs) associated with the computational operation of the Byesian Regularization (BR), referred to as ENNs-BR. All the outcomes of the DCOC model are conducted by comparing their performance. For the statistical study, the data of the considered model is partitioned into 75% and 15% in two parts. Finally, the results for Performance, Training state, Error histogram, Regression, and fit are graphically illustrated.

***K*eywords** Fractional order Caputo Cancer model · Euler method · neural networks · Byesian Regularization · Training state.

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BILINEAR MULTIPLIERS ON ORLICZ SPACES ON LOCALLY COMPACT GROUPS

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ABSTRACT

In this work, we generalize the theory of bilinear multipliers acting on Orlicz spaces from $\mathbf{R}^n$ to locally compact abelian groups G. We focus on describing these bilinear multipliers from the point of view of abstract harmonic analysis. We obtain separate necessary and sufficient conditions for the existence and boundedness of such bilinear multipliers.

***K*eywords** Bilinear multipliers · Orlicz spaces · Locally compact abelian groups · Boyd Indices

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APPLICATION OF SUPERVISED MACHINE LEARNING ALGORITHMS TO IDENTIFY THE PREVALENCE AND DETERMINANTS OF SPONTANEOUS ABORTION AMONG EVER-MARRIED WOMEN IN SOMALILAND: INSIGHTS FROM SLDHS DATA 2020

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ABSTRACT

This study examined the prevalence and determinants of spontaneous abortion among ever-married women in Somaliland using the Somaliland Demographic Health Survey 2020. The results showed that 95.85% of respondents had unintended abortions, while 4.1% had not experienced abortions. The results were presented using Chi-square with a 95% confidence interval (CI) and a p-value $\leq$ 0.05. The analysis indicates that abortion rates vary across regions in Somaliland. The Waqooyi Galbeed region has a statistically significant association with abortion compared to other regions AOR=2.4, p-value=0.002. Additionally, individuals in the second wealth index category are more likely to have abortions AOR=2.01, p−value=0.012. The study also found that difficulties in accessing medical help are significantly related to abortion AOR=1.6, p−value=0.005, and individuals with more than five children are more likely to have spontaneous abortions, showing a statistically significant association AOR=1.9, p−value=0.003. Five machine learning models were used to analyze predictors of spontaneous abortion, with Random Forest and KNN being the most accurate models, achieving accuracy rates of 95.9% and 95.5% respectively.

***K*eywords** Spontaneous abortion · machine learning · logistic regression· prediction

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A TECHNIQUE FOR DERIVING $q-$ANALOGUES OF CERTAIN SPECIAL MATRICES

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ABSTRACT

In matrix theory, there are numerous useful and special matrices. One of the most well-known and extensively studied matrices among them is undoubtedly the Pascal matrix. The algebraic approach based on Pascal matrices holds significant importance across various mathematical fields including algebraic geometry, optimization, matrix theory, and combinatorics. Several authors have extensively introduced and studied specific generalizations of Pascal matrices in detail. Matrix representations of sequences of various polynomials is also intriguing. We introduce a comprehensive approach for constructing $q-$analogues of some special matrices including Pascal matrix and matrices related with some polynomials. Using this method, we find explicit expressions for linear algebraic properties of matrices such as products, inverses, and powers, as well as specific factorization formulas.

***K*eywords** Pascal matrix · Modified Hermite matrix · q-Integers · Matrix factorizations · Matrix inversion

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BOUNDARY VALUE PROBLEMS IN POLY-HARDY CLASS

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ABSTRACT

In this talk, we discuss the boundary value problems in poly-Hardy class. Firstly we mention about the Dirichlet problem in the unit disc. Then we investigate the functions in Hardy class and their properties over unit disc. Lastly we obtain the solution of Dirichlet problem for higher-order Cauchy-Riemann equation in poly-Hardy class.

***K*eywords** Boundary value problems · poly-Hardy class · unit disc

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GLOBAL DYNAMICS FOR A MODEL WITH STAGE STRUCTURE AND SPATIAL MOVEMENT

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ABSTRACT

Spatial diffusion and insecticide resistance, is a pest survival process. To better understand these traits, we discuss a structured model with two life stages, juveniles and reproducing adults. We present a systematic study using monotone systems theory. Using this model, we aim to understand the main characteristics leading to persistence or extinction of the insect pest population. We establish global stability results. Numerical simulations provide some interesting insights on the dynamics of the pest population. The dynamics is described in terms of a threshold value obtained by the spectral radius of the Poincaré operator of the linearized problem around the extinction equilibrium.

***K*eywords** aggregated model · cooperative systems · Insects · slow fast dynamics

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ON SOLUTIONS OF A THIRD ORDER LINEAR DIFFERENCE EQUATION WITH VARIABLE COEFFICIENTS. APPLICATIONS.

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ABSTRACT

In this article, we explore the determinantal and combinatorial approaches to third order linear difference equations with variable coefficients. We present and demonstrate some of its properties. Furthermore, as an application, we examine the generalized Jacobsthal sequence and the sequence of powers of two related to the Rogers-Ramanujam type identities establishing new results for these polynomial sequences.

The sequence of Jacobsthal numbers is usually represented by the recurrence relation given by $J_n = J_{n-1} + 2J_{n-2}$, with initial conditions $J_0 = 0$ and $J_1 = 1$. Applications of this recurrence can be seen in several works, such as the data in [1, 8, 5] where combinatorial and analytical interpretations are offered for this sequence of integers. Another much studied sequence is given by the recurrence relation $P_n = 2P_{n-1}$, with $P_0 = 1$. It refers to the powers of the integer 2, since $P_n = 2^n$ for all $n \geq 0$. This sequence is closely linked to Set Theory, as it represents the number of subsets of a set with n elements. Furthermore, several other interpretations are associated with this sequence, as can be seen in [7]. More specifically, as applications, we explore the q-analogues of the sequences J_n and P_n, denoted here by $\{J_n(q)\}_{n\geq 0}$ and $\{P_n(q)\}_{n\geq 0}$.

A q-analogue is a polynomial in the indeterminate q that generalizes certain sequences of integers. The q-Jacobsthal $J_n(q)$ ($q-$ analogue for J_n) satisfies the following recurrence relation of order 3 with variable coefficients

$$J_{n+2}(q) = J_{n+1}(q) + (q + q^n)J_n(q) + (q^n - q)J_{n-1}(q) \tag{1}$$

with initial conditions $J_0(q) = 1 = J_1(q)$ and $J_2(q) = 1 + q + q^2$. This generalization was introduced by Santos in [3] and worked on by Craveiro in [2]. Note that, when $q = 1$, then $J_{n+2}(1)$ is nothing more than the sequence of Jacobsthal numbers.

The sequence $q-$powers of 2 was introduced by Sills in [4] and satisfies the following difference equation

$$P_{n+2}(q) = (1 + q)P_{n+1}(q) + (q^{n+3} - q)P_n(q) + (q^{2n+4} - q^{n+3})P_{n-1}(q), \tag{2}$$

with $P_0(q) = 1, P_1(q) = 1 + q$ and $P_2(q) = 1 + q + q^2 + q^3$. For $q = 1$ we have $P_{n+2}(1) = 2^{n+2}$.

The sequences given by Equations (1) and (2) offer interesting combinatorial interpretations for some series-product identities, listed by Slater in [6] and also by Sills in [4]. By satisfying difference equations, we can use various techniques and methods to find new expressions for the $q-$analogues.

***K*eywords** Third order linear difference equations · Determinantal approach · Combinatorial approach · Jacobsthal sequence · Powers of two sequence · Rogers-Ramanujan Identities

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ON TRANSLATION SURFACES IN MULTIPLICATIVE CALCULUS

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ABSTRACT

In this study, generating curves of translation surfaces are examined with some special curve pairs in multiplicative calculus. With the results obtained from these pairings, the developable and minimal translation surfaces are characterized by using multiplicative calculus in Euclidean 3-space. Furthermore, the surface curvatures of the translation surface are obtained. For a better understanding of the results, examples are given and their drawings are made with the help of Mathematica.

***K*eywords** Translation surface · Multiplicative calculus · Special curve pairs

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NUMERICAL METHODS

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ABSTRACT

This work is based on the theoretical framework of Godino and collaborators' Ontosemiotic Approach (OSA), analyzing the didactic adequacy of a didactic intervention in the learning of the themeedson.cruz@ifpa.edu.br "Curve fitting and interpolation" in a class of 4th-year students of higher education in Mathematics (14 boys and two girls) of the Federal Institute of Education of Pará —IFPA, Campus Belém (Brazil).

In the first stage, the students' errors and difficulties (semiotic conflicts) were identified in works described in the literature, and an initial didactic intervention was remote teaching. In the second stage, a didactic intervention was implemented using a script designed for learning numerical calculation with technologies (free computer software - Octave, spreadsheet, and free Android apps for cell phones and tablets).

This work presents an implementation of the didactic intervention in a numerical calculus course. A qualitative perspective and an interpretive paradigm were used to detect the didactic intervention's didactical suitability.

Based on the students' results analysis, the didactic suitability of the didactic intervention carried out in the Mathematics degree class was characterized. It was concluded that there was high didactic suitability for the epistemic (problem situations, language, rules, arguments, and relationships), emotional (attitudes, affections, and motivations), and interactional (dialogues, interactions, and communications) components, medium/high didactic suitability for the cognitive (proximities) and mediational (resources - guiding task and technologies, as well as time) components and, finally, the didactic suitability for the ecological facet (adequacy of the curriculum, in particular) was medium.

In short, the overall didactic suitability of the "Curve fitting and interpolation" classes was considered high since they managed to overcome many semiotic conflicts detected in this topic, both in the previous remote intervention and the literature.

The epistemological and cognitive components were balanced in these presential classes. In addition, the face-to-face environment favored observations of the students' gestures (emotions and attitudes) in these presential classes, access to technological tools, the intense dialogue between students and student/teacher in explanations and carrying out tasks (individual and group) contributed to improvements in the affective, interactional and emotional components.

We also observed the good relationship and interest of the students in the class with the objects of the subject under study and their technological applications (scientific and technological), which highlighted the mediational facet and, indirectly, boosted other components.

***K*eywords** Numerical Analysis · Technology · Ontosemiotic approach · Didactic suitability

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A New Optimization Problem Constituting the Best Rank Order

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Abstract

Rank order is a scaling technique commonly used in social sciences. In this approach, participants are expected to order given stimuli based on various criteria such as importance, usefulness, necessity, or preference. For example, in a study conducted to determine the purposes for using the Internet, observers are asked to rank the predefined stimuli such as Researching, Shopping, Social Media, Listening to Music, Watching Videos, and Electronic Banking according to the frequency of their use. In this example, although observers were asked to rank 6 stimuli according to a criterion, they were forced to make a distinction. It is more difficult to make this distinction in rankings that are expected to be made considering more than one criterion, but the observer is forced to make this distinction. In this method, the validity of rank order scaling is very high, as observers have to make as many discriminations as possible between the stimuli.

The data from observers is then utilized to establish a rank order that best represents the information. Although there is a statistical-based ranking method [4, 6] in the literature, there is no optimization-based ranking method. Optimization theory sometimes deals with finding the center of data classes, which is called the data classification [1, 3] or the best curve representing the data, which is called the curve fitting [2, 5]. Considering the similarities of the ranking order with these two problems, an optimization problem will be proposed to find the mentioned rank order. The reasons for searching for a new ranking order method are that different stimuli can share the same order in the statistics-based ranking order scale, and a stimulus that has never been in the first place can be placed in the first place in the best order. The proposed new method is planned to overcome these problems.

***K*eywords** Rank Order · Optimization · Machine Learning

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A New Operator Class Associated with L-weakly and M-weakly Compact Operators

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Abstract

The operator classes L-weakly and M-weakly compact were introduced by P. Meyer-Nieberg. It was established that these classes are subsets of weakly compact operators, yet they do not imply compactness, nor do compact operators necessarily fall into these categories. Specifically, a non-empty bounded subset A of a Banach lattice E is defined as L-weakly compact if $\|x_n\| \to 0$ as $n \to \infty$ for every disjoint sequence (x_n) contained within the solid hull of A. A bounded linear operator T from a Banach space X to E is said to be L-weakly compact if $T(B_X)$ is L-weakly compact in E, where B_X represents the closed unit ball in X. Conversely, a bounded linear operator from E to X is M-weakly compact if $\|Tx_n\| \to 0$ as $n \to \infty$ for every disjoint sequence (x_n) in B_E.

In contrast to the complexities of analyzing weakly compact operators within a general Banach lattice framework, the subclasses of L-weakly and M-weakly compact operators, which are specific types of weakly compact operators, exhibit a range of intriguing properties. Extensive literature addresses the interactions between L-weakly and M-weakly compact operators and their connections with weakly compact operators and other operator classes. Although this operator class does not exist, it has been established that the class of regular L-weakly and regular M-weakly compact operators is in the form of a Banach lattice and possesses many attractive ordered properties.

Recently, several new classes of operators related to L-weakly and M-weakly compact operators have been introduced, along with their associated properties. These include LW-compact operators, almost L- and M-weakly compact operators, null almost L-weakly compact operators, and un L-weakly and un M-weakly compact operators. These operator classes are generalized new classes of L-weakly and M-weakly compact operators. In this study, we have introduced a new class of operators termed "LM-compact operators," which is defined in relation to L-weakly and M-weakly compact operators. It was found that this new class of operators retains some of the ordered properties of the original class, while lacking others. Additionally, comparisons with compact and weakly compact operator classes, as well as some of its algebraic properties and duality properties, were also investigated.

***K*eywords** Banach lattice · L-weakly compact · M-weakly compact

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WIJSMAN CONVERGENCE OF SEQUENCES OF SOFT SETS

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ABSTRACT

The concept of convergence is one of the most crucial in mathematics, intricately tied to the mathematical structures in which it is defined and facilitating a deeper understanding of these structures. There exist numerous types of convergence, both topological and non-topological, within the same mathematical framework that can be compared to one another. One such convergence type is Wijsman convergence, which defines the convergence of sequences of closed sets to a closed set, analogous to how convergence of sequences of points is defined in a metric s pace. Initially defined by R. A. Wijsman, this concept had been used previously by Z. Frolík and Hausdorff for appropriate metric spaces. Wijsman convergence, defined with respect to a metric, is a variation of Hausdorff convergence. It establishes a topology on the family of closed sets of a metric space, where this topology can vary even for uniformly equivalent metrics. Another focus of our study is on soft sets, introduced by Molodtsov as a method to handle uncertainties where traditional set concepts are inadequate. While there are studies on the topological convergence of sequences of soft points in topological spaces formed by soft sets, there has been no specific investigation into the convergence of sequences of soft sets. In parallel with the classical metric definition, the concept of soft metric is defined on the set of soft points, which is a special case of soft sets. Hence, considering Wijsman convergence for sequences of closed soft sets using this soft metric concept is natural. Consequently, in this study, we define and explore the Wijsman convergence of sequences of closed soft sets to a closed soft set in a soft metric space, investigating their general properties. We introduce the notion of soft Wijsman Cauchy sequences and explore their relationship with soft Wijsman convergent sequences. Additionally, we observe the existence of other convergence types in soft metric spaces and their relationships with Wijsman convergence, aiming to contribute to the literature by integrating soft set theory with Wijsman convergence concepts.

Keywords Soft set · Soft point · Wijsman convergence

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IMPROVED COMPUTATIONAL TECHNIQUES FOR HEAT SOURCES LOCALIZATION

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ABSTRACT

We consider the inverse heat source problem, which consists in determining the position and intensity of unknown heat sources within a given domain based on observed temperature distributions. The inverse heat source problem has demonstrated significant potential for applications in fields such as thermal management [1], pollution detection [2], and in general in different engineering problems [3]. In particular, we are interested in the numerical solution of this problem for an application in the living sector [4], so the domain under consideration is a three-dimensional region. In this context, we propose a novel approach that leverages Green's function methods to represent the solution of the heat conduction problem under study. This, in turn, allows us to construct a Volterra equation of the first kind, to formulate the inverse problem.

The formulation of the representation formula is based on the solution of the following one-dimensional problem for the heat equation with appropriate boundary conditions and initial temperature distribution:

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) - \alpha \frac{\partial u}{\partial x^2}(x,t) = f(x,t), & x \in \Omega,\ t \in [0,T], \\ \frac{\partial u}{\partial x}(0,t) = 0, & t \in [0,T], \\ \frac{\partial u}{\partial x}(L,t) = 0, & t \in [0,T], \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where we consider $\Omega = [0, L]$ as the spatial domain and $[0, T] \subset \mathbb{R}$ as the time interval; $\alpha \in \mathbb{R}$, $\alpha > 0$, is a constant term representing the thermal diffusivity, $u_0(x)$ is the function describing the initial temperature, i.e., the temperature at time $t = 0$, and $f(x, t)$ is the function representing the source term. The source term is modeled as a sum of Gaussian functions, which provided computational advantages in our case study, [5]. From standard arguments on heat equation theory, the solution of problem (1) can be written in integral form by involving the Green function associated with problem (1), see [6] for details.

From the solution of problem (1), we extend our methodology to the three-dimensional problem:

$$\begin{cases} \frac{\partial u}{\partial t}(\mathbf{x},t) - \alpha \Delta u(\mathbf{x},t) = \phi(\mathbf{x},t), & \mathbf{x} \in B,\ t \in [0,T], \\ \frac{\partial u}{\partial \hat{n}}(\mathbf{x},t) = 0, & \mathbf{x} \in \partial B,\ t \in [0,T], \\ u(\mathbf{x},0) = U_0(\mathbf{x}), & \mathbf{x} \in B; \end{cases} \tag{2}$$

where the spatial domain is $B = [0, L_x] \times [0, L_y] \times [0, L_z] \subset \mathbb{R}^3$, $L_x > 0$, $L_y > 0$, $L_z > 0$, and ∂B is the boundary of the domain B. Moreover, $\hat{n}(\mathbf{x})$ represents the unit outward normal at $\mathbf{x} \in \partial B$, Δ is the Laplacian operator, $\phi(\mathbf{x}, t)$ is the three-dimensional function describing the source term and $U_0(\mathbf{x})$ is the function modeling the initial temperature. An integral formulation of the solution $u(\mathbf{x}, t; U_0, \phi)$ of problem (2) is obtained [4]. This involves the functions $U_0(\mathbf{x})$, $\phi(\mathbf{x}, t)$ and the Green function associated to problem (2).

We study the following inverse problem: determine the source intensities that match some temperature measurements at specific points within the domain at precise different times. More specifically,

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given two positive integers I and J, we define $\mu_i(t_j)$,, $i = 1, 2, \ldots, I$, $j = 1, 2, \ldots, J+1$, as the temperature measurements at known points $\mathbf{x}_i \in B$, $i = 1, 2, \ldots, I$ at times t_j, $j = 1, 2, \ldots, J+1$, we want to compute the source intensity $\phi(\mathbf{x}, t)$ such that $u(\mathbf{x}_i, t_j; U_0, \phi) = \mu_i(t_j)$, $i = 1, 2, \ldots, I$, $j = 1, 2, \ldots, J+1$. Supposing to know U_0, this is an integral equation for the source intensity function. We present some results obtained with an improved numerical solution of this integral equation. The main novelties are related to the use of the midpoint rule [7] for the numerical quadrature and a proper use of the measurements in different times to obtain a more efficient and stable discretization scheme. More precisely, the numerical quadrature leads to the definition of a linear system that, before being numerically solved, is modified by applying the Tikhonov regularization technique [8] to find a stable solution with non-negative components through a constrained minimization problem. Moreover, an iterative approach leads to solving the problem for subsequent subintervals $[0, t_j]$ of $[0, T]$ instead of attempting to solve it for the entire time interval at once. In particular, for a given $p < J + 1$, at the generic step k of the proposed iterative procedure we utilize the values of source intensities already computed at times t_j, $j = 1, \ldots, j_{k-p}$, U_0, ϕ, to compute the unknown values of the source intensities at times t_j, $j = j_{k-p}, \ldots, j_k$. So, we retain as variables only the source at the last p measurement times. Finally, some numerical results will be presented to illustrate the efficacy of our proposed method.

***K*eywords** Inverse source problem · Volterra integral equation · Living sector · Green function method

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DIAGONALIZATION OF 4TH ORDER MATRICES REPRESENTING A QUATERNION

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ABSTRACT

This paper explores the diagonalization of 4th-order matrices representing quaternions, focusing on two distinct matrix representations: the left representation and the right representation. The study details a comprehensive diagonalization algorithm, including the steps necessary to determine eigenvalues and eigenvectors for quaternion matrices. By using specific examples, the paper demonstrates the application of this algorithm, providing a clearer understanding of the diagonalization process. The research highlights the importance of these matrix representations in simplifying the analysis of quaternion structures, emphasizing their utility in streamlining complex quaternion computations and improving their interpretability.

Keywords Quaternions · Diagonalization 4th-order matrices · Eigenvalues-Eigenvectors

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ON THE CLASSIFICATION OF FINITE GROUPS

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ABSTRACT

Finite groups are algebraic objects fundamental to the study of symmetry, and therefore widely applicable to most branches of mathematics concerned with finite objects. The comprehensive classification of finite simple groups is attributed to Daniel Gorenstein in 1983. However, it was not declared complete until revisions were made by Aschbacher and Smith correcting the proof, which initially totaled over 10000 pages. In this paper we will present the classification of finite groups. We begin by discussing basic definitions and theorems in group theory. We proceed to describe the groups with 4 and 6 elements and to determine the non-isomorphic groups with 8, 9, 10 and 12 elements.

Keywords Finite groups · Order of group · Classification of finite groups

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OPEN ACCESS FISHERIES MODELS CONSIDERING DEPENSATORY GROWTH FUNCTIONS IN THE EXPLOITED RESOURCE

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ABSTRACT

In this work we study two classes of continuous-time deterministic bioeconomic models that represent the exploitation of a resource in an open-access fishery, assuming that:

i) The population growth of the exploited biomass is affected by the phenomenon called *depensation* or *Allee effect* [1].
ii) The resource harvesting function is based on the Schaefer hypothesis, that is, the catch per unit effort (CPUE) is proportional to the biomass.
iii) Fishing effort varies continuously over time as does population size as postulated in Smith's bioeconomic model [2].

Besides, in the first model, we will use the *Pella and Tomlinson growth function* [3] an asymmetric curve describing the exploited biomass growth.

A modification of the Smith model is considered, where the exploited resource is affected by depensation or the Allee effect. We will describe this effect with two different growth functions, obtaining that autonomous bidimensional ordinary differential systems represent the models.

We conjecture the dynamical behavior of the models to be analyzed has marked differences, particularly on the number of limit cycles surrounding a positive equilibrium point.

***K*eywords** Bioeconomic model · depensation · CPUE · stability · limit cycles

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MODELLING MEDFLY PEST MANAGEMENT

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ABSTRACT

The Mediterranean fruit fly Ceratitis capitata Wiedemann is a very important pest of fruit crops. Originally from the African continent, it is now distributed worldwide and causes great losses in the quantity and quality of fruit, making its trade difficult or impossible. One form of pest control is the so-called "sterile insect technique", which is widely used because it is environmentally friendly. The introduction of sterilized insects into the agricultural system seeks to reduce the pest's reproduction rate with the consequent decrease in population size. The mathematical modelling of this problem is presented in this paper in three stages. First, the following theoretical model is proposed for the state of the pest in the absence of control:

$$\begin{aligned} \frac{dx}{dt} &= \varphi\nu y - (\alpha+\gamma)x - cx^2 \\ \frac{dy}{dt} &= p\alpha x - \mu_h y - \beta y^2 \\ \frac{dz}{dt} &= (1-p)\gamma x - \mu_m z - \delta z^2 \end{aligned}$$

where x, y and z are the states variables corresponding to count of juvenils (eggs, larvae and pupae), count of adult females and count of adult males. The positive real parameters are $\varphi, \alpha, \gamma, c, \beta, \delta, \mu_h, \mu_m, \nu$ and p.

In a second stage an empirical rule is introduced and consecuently the model obtained is a theoretical-empirical hybrid wich is simpler than the original having two differential equations: one for the female's growth rate and the other for the male's growth rate.

In last stage, control by the sterile insects is included.

For the models in each state stability conditions of some equilibrium points as well as pest persistence or pest eradication conditions are presented and justified in this work wich is developed in an field of confluence of Mathematics, Biology, Ecology and Agronomic Engineering.

***K*eywords** Mediterranean fly · Sterile Insect Technique · Dynamical Systems in Ecology

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BINET-FIBONACCI CALCULUS AND N = 2 SUPERSYMMETRIC GOLDEN QUANTUM OSCILLATOR

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ABSTRACT

The Binet-Fibonacci calculus, as $\varphi\varphi'$ - two base quantum calculus, relates Fibonacci derivative with Binet formula of Fibonacci number operator, acting in Fock space of quantum states. It provides a tool to study the Golden oscillator with energy spectrum in form of Fibonacci numbers. Here we generalize this model to supersymmetric number operator and corresponding Binet formula for supersymmetric Fibonacci operator $\mathcal{F}_{\mathcal{N}}$. It determines the Hamiltonian of supersymmetric Golden oscillator, acting in $H_f \otimes H_F$ - fermion-boson Hilbert space and belonging to $N = 2$ supersymmetric algebra. Trace on fermions of this model reduces the Hamiltonian to the Golden oscillator. The eigenstates of the super Fibonacci number are double degenerate and can be characterized by a point of the super-Bloch sphere. By the supersymmetric Fibonacci annihilation operator, we construct the coherent states as eigenstates of this operator. Entanglement of fermions with bosons in these states is calculated by the concurrence, represented by the Gram determinant and Fibonacci exponential functions. These functions have been appeared as descriptive for inner product of the Golden coherent states in Fock-Bargmann representation. The reference state, coming from the limit $\alpha \to 0$ and corresponding von Neumann entropy, measuring fermion-boson entanglement, are characterized by the Golden ratio.

***K*eywords** Fibonacci numbers · Golden Ratio · supersymmetry · coherent states · Golden oscillator

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GEOMETRY AND ENTANGLEMENT OF SUPER-QUBIT QUANTUM STATES

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ABSTRACT

We introduce the super-qubit quantum state, determined by superposition of the zero and the one super-particle states, which can be represented by points on the super-Bloch sphere. In contrast to the one qubit case, the one super-particle state is characterized by points in extended complex plain, equivalent to another super-Bloch sphere. Then, geometrically, the super-qubit quantum state is represented by two unit spheres, or the direct product of two Bloch spheres. By using the displacement operator, acting on the super-qubit state as the reference state, we construct the super-coherent states, becoming eigenstates of the super-annihilation operator, and characterized by three complex numbers, the displacement parameter and stereographic projections of two super-Bloch spheres. The states are fermion-boson entangled, and the concurrence of states is the product of two concurrences, corresponding to two Bloch spheres. We show geometrical meaning of concurrence as distance from point-state on the sphere to vertical axes - the radius of circle at horizontal plane through the point-state. Then, probabilities of collapse to the north pole state and to the south pole state are equal to half-distances from vertical coordinate of the state to corresponding points at the poles. For complimentary fermion number operator, we get the complimentary super-qubit state and corresponding super-coherent state, as eigenstate of transposed super-annihilation operator.

***K*eywords** qubit · super-qubit · super-coherent state · super-Bloch sphere · entanglement

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CONTROL ON THE BOUNDARY OF UNSTEADY NATURAL CONVECTIVE SLIP FLOW OF REACTIVE VISCOUS FLUID

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ABSTRACT

We consider the optimal control of unsteady natural convective flow of reactive viscous fluid with heat transfer. It is assumed that Newton's law follows the heat transfer within an exothermic reaction under Arrhenius kinetics and Navier slip condition on the lower surface of the channel. The flow is examined in a vertical channel formed by two infinite vertical parallel plates having a distance H between them. Time-dependent natural convective slip flow of reactive viscous fluid flow and heat transfer equations in [1] are solved in a unit interval using Galerkin-Finite Element Method (FEM) with quadratic finite elements in space and the implicit Euler method in time. The direct solutions are obtained for testing various values of the problem parameters, the Biot number, the Frank Kamenetskii parameter, the Navier slip parameter, and the computation of the skin friction and the Nusselt number Nu.

The optimal control problem is designed for the momentum and energy equations to derive the fluid-prescribed velocity and temperature profiles by defining controls on the boundary of the domain twofold: (a) controls are assigned as the time-dependent functions in the boundary conditions representing the slip velocity and the heat transfer rate. (b) controls are formulated as the parameters in the boundary conditions such as slip length and Biot number. Following a discretize-then-optimise approach to the control problem, optimisation is performed by the SLSQP (Sequential Least Squares Programming) algorithm, a subroutine of SciPy. Numerically simulated results show that the proposed approach successfully drives the flow to prescribed velocity and temperature profiles.

***K*eywords** natural convection · optimal control · finite element method

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NEW GRÖNWALL-FREDHOLM TYPE INEQUALITY WITH APPLICATION TO A LARGE CLASS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT

In this work, we consider a Ψ-Caputo fractional Newton's Second Law of motion involving the ψ-Caputo operator of order $\alpha \in (1, 2]$. We prove the existence and uniqueness of solutions for different classes of force functions f acting on a specific object in motion. We introduced a new Grönwall-Fredholm type inequality ; a generalization of well-known Pachpatte's result, but under a weaker condition, that will help to prove Ulam-Hyers-Rassias stability $\mathbb{UHR}$ and generalized Ulam-Hyers-Rassias $\mathbb{GUHR}$ stability of the solutions for a large class of fractional differential equations, mainly the cases where the solutions related to a Green function. In addition, we replace the well-known used condition;

- μ is a continuous function over $[a, b]$,
- μ is an increasing function over $[a, b]$,
- There exists a positive constant γ_μ such that
$$ {}^{f}I^{\alpha}_{a+}\mu(t) \leq \gamma_\mu \mu(t), \ \forall t \in [a, b]. $$
where ${}^{f}I^{\alpha}_{a+}$ denote a fractional integral.

by a new condition that is considerably simpler and weaker ; μ only a positive continuous function with $\inf_{s\in[a,b]} \mu(s) > 0$. Finally, we provide a example to establish our theoretical results.

Keywords Fractional differential equations · ψ-Caputo derivative · Grönwall type inequality · Ulam-Hyers stability

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A QUADRATIC PROGRAMMING MODEL FOR INPATIENTS OPERATING ROOM SURGERIES SCHEDULING BASED ON RESOURCES AVAILABILITY

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ABSTRACT

Scheduling inpatients' surgeries is a multifaceted scheduling problem. Variety of factors such as availability of operating rooms, surgeons' or doctors' available time, working hours, the number of operations in one planned period and unfavourable surgery hours. can make it very challenging to come up with the optimal schedule for the surgeries. Studies on inpatients' surgeries scheduling problems are still lacking, while those solved using Nonlinear Programming model are considered more complex compared to the Linear Programming model. This study concerns with solving inpatients' operations scheduling using the Quadratic Programming (QP) model. The main objectives of this study are to propose an enhanced formulation of the QP model that minimizes the total overtime and idle time of any operating room to ensure full utilization of the operating rooms, and to solve the model using the exact and heuristic approaches of MATLAB quadprog, and to analyze doctors' schedule based on the model's solution. This study employed data from a selected past study. Based on the solution obtained, the total idle and overtime hours is minimized to only 64.01 hours. Meanwhile, no doctor has been scheduled to operate on more than one inpatient in the same day. In addition, the operation rooms have been scheduled evenly with 25 inpatients' operations for each room. By reducing the total overtime and idle time will enables reduced expenses and consequently costs savings and increased revenue to the hospital and improve the quality of healthcare. Meanwhile, systematic scheduling offers more effective schedule that reduces long waiting time for surgery and length of stay at hospital for inpatients, optimizes the use of the operating rooms, and more organized and practical schedules for the doctors. Moreover, the schedule can be achieved in a timely manner by solving the model.

***K*eywords** Inpatients Surgery Scheduling · Planning and Scheduling · Operating Room / Theater · Quadratic Programming Model · Total Idle and Overtime

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V International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2024)

ICMASE 2024

EFFECTS OF AUTONOMOUS VEHICLES ON PARTICULATE MATTER

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ABSTRACT

A significant source of air pollution is the emissions due to vehicular traffic. The latter is affected by features of vehicles and driving habits which can exacerbate emissions. On the other side, air pollutants have impact on human health and life quality in general. A strong correlation was found between the presence of particulate matter and indicators such as health and school performance.
This work aims to analyze the potential positive effect of autonomy on the reduction of pollutants. The approach is based on comparing particulate matter emissions linked to different traffic conditions, which in turn are regulated via the insertion of a small number of autonomous vehicles in bulk traffic. Traffic data are gathered from an experiment exhibiting the appearance of stop-and-go waves for a fleet of twenty vehicles traveling on a ring road, and the subsequent wave dampening via longitudinal controls implemented on a single autonomous vehicle. It is shown how the wave dampening causes a significant reduction on particulate matter emission. Then, using the principal chemical reactions in atmosphere, a system of differential equations for the concentrations of the principal pollutants at street level is defined and numerically solved. This in turn provides estimates on the presence of various air pollutants due to traffic emissions and their reduction thanks to the traffic smoothing via autonomous vehicles.

***K*eywords** road traffic · emissions · particulate matter · differential equations

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GEOMETRIC AND VARIATIONAL ANALYSIS OF HYPERELASTIC STRIPS WITH TIMELIKE BASE CURVES

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ABSTRACT

The developable ruled surfaces whose modified Darboux vector is determined according to the unit tangent vector field and the binormal vector field of the reference curve are known as rectifying strips. The elastic behavior of rectifying strips is studied by minimizing the functional defined by Sadowsky in 1930. A necessary condition for a rectifying strip to be elastic is that the reference curve is a critical point of the Sadowsk functional. The fact that the Sadowsky functional is proportional to the Wilmore functional provides a different perspective on Willmore surfaces, which are frequently studied in geometry. Recent studies have shown that the p-Willmore functional, which is a more general case of the Willmore functional and is known to have applications in biology and quantum mechanics, and the Sadowsky-type functional, which is a more general case of the Sadowsky functional, are proportional. Also, according to recent studies, the rectifying strips formed by the critical points of the Sadowsky type functional are introduced as hyperelastic strips (or p-elastic strips). It is noteworthy that the Euler Lagrange equations characterizing hyperelastic strips are used to determine geometric conservation laws with the help of conservative quantities defined with the help of Euclidean motions. In this paper, we define the modified Sadowsky-type functional in three-dimensional Minkowski space and using timelike reference curves, derive the Euler Lagrange equations that characterize the critical points of the modified Sadowsky-type functional. We denote hyperelastic strips with timelike base curves (or p-elastic strips with timelike base curves) as the rectifying strips whose base curves are solutions for these Euler Lagrange equations. We establish a connection between hyperelastic curves and hyperelastic strips with timelike base curves in the case of torsion-free. We obtain conserved quantities for hyperelastic strips with timelike base curves for Minkowski spaces with Poincaré isometry groups. Then, we get the first and second conservation laws of hyperelastic strips with timelike base curves.

Keywords Conservation laws · Hyperelastic strips with timelike base curves · Sadowsky-type functional · Variational calculus.

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ON GEOMETRY OF ASSOCIATED CURVES OF TIMELIKE MAGNETIC CURVES

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ABSTRACT

The study of magnetic curves and vector fields represents a significant area of research in both the fields of physics and mathematics. A fundamental property of any magnetic vector field is that it is a divergence-free vector field in three spaces. Magnetic vector fields are generated by electric currents and changing electric fields, and charged particles moving within these fields are influenced by magnetic forces. The paths traced by these particles, known as magnetic trajectories, are referred to as magnetic curves. The study of magnetic curves is crucial for both theoretical physics and engineering applications, including the analysis of electromagnetism, plasma physics, and magnetic materials. Also, the issue of investigating magnetic trajectories initially appears to be a purely physical problem, recent studies have demonstrated that the characterization of magnetic flow in a magnetic field has introduced a variational perspective that is more geometrical in nature. In particular, magnetic curves have been developed through the application of techniques derived from differential geometry and methods of the calculus of variation, from basic spaces to manifolds. The tangent, normal, and binormal vectors characterize the kinematic and geometric aspects of a particle's motion, influencing the path of a charged particle in a magnetic field. Additionally, the time dimension plays a role in shaping its trajectory. In this paper we first investigate whether there are Mannheim, Bertrand and involute-evolute curve pairs for a timelike N-magnetic curve in 3-dimensional Minkowski space. The relationships between this curve and the curve pairs are obtained. In this way some geometric properties between the curves are derived. Similar procedures are carried out for timelike B-magnetic curves and important results are obtained.

***K*eywords** Killing vector fields · Lorentz force equation · Magnetic trajectory.

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INCOMPLETE FERMATEAN FUZZY PREFERENCE RELATIONS AND SOCIAL NETWORK DECISION-MAKING: THE INVESTMENT PORTFOLIO SELECTION APPLICATION

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ABSTRACT

The modern portfolio theory is a valuable technique for choosing investments to optimize total returns while maintaining a manageable amount of risk. An investment portfolio is built to optimize expected return given a specific level of risk using this mathematical framework. Problems with portfolio selection are ideally suited for multi-attribute decision-making algorithms. Within the multi-attribute decision-making paradigm, complicated subjective preferences and diversified financial indices influence investment decisions.

There may be cases where experts do not have in-depth knowledge of the problem to be solved in decision-making problems. In such cases, experts may fail to express their views on certain aspects of the problem, resulting in incomplete preferences, in which some preference values are not provided or are missing. A new model for group decision-making methods will be given in which experts' preferences can be expressed as incomplete Fermatean fuzzy preference relations. The additive-consistency property guides this model and only uses the expert's preference values. An additive consistency definition characterized by a Fermatean fuzzy priority vector has been given. The additive consistency property is also used to measure the level of consistency of the information provided by the experts. The proposed additive consistency definition's property and a model for obtaining missing judgments in incomplete Fermatean fuzzy preference relations will be provided. A method for adjusting the inconsistency for Fermatean fuzzy preference relations, a model for obtaining the priority vector, and a method for increasing the consensus degrees of Fermatean fuzzy preference relations will be used.

In recent years, the development of information technology has enabled social networks to be online communication platforms for individuals to exchange messages and share information. This enhanced communication environment leads to a new format of group decision-making that acknowledges the influence of the social relationships among experts on the decision process and results, i.e., social network group decision-making.

Our research has developed a social network group decision-making framework using incomplete Fermatean fuzzy preference relations to address portfolio investment selection issues, especially when multiple decision-makers are involved.

***K*eywords** Fermatean fuzzy set · incomplete preference relations · social network · investment portfolio selection · decision-making

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FAULT INJECTION ATTACKS AGAINST RSA-CRT DIGITAL SIGNATURE

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ABSTRACT

It is known that Cryptology has as objective to guarantee the confidentiality, integrity and authentication of information when it is stored or transmitted. Asymmetric cryptosystems are among the most used methods for encrypting the information, and one of their main applications is the digital signature. In this type of cryptosystems each user has two different keys: the public and the private one. The first one is publicly known and permits any user to encrypt information or verify the digital signature of the owner of the key; whereas the private key, which is secret, is used by the owner for decrypting the encrypted information or elaborating his digital signature.

The RSA cryptosystem is considered to be mathematically secure against the integer factorization methods, but there are other attacks whose objective is to obtain the private key [1]. In fact, Side Channel Attacks rely on analyzing certain information obtained from the device where an implementation of the algorithm runs; for example, measuring the execution time or the power consumption required by the algorithm, the electromagnetic field generated by the device, etc. On the other hand, Fault Injection (FI) Attacks deliberately induce a fault in the execution of the algorithm to get a wrong output, so that such output allows the attacker to make guesses based on the comparison of different results.

In this work, we explain the mathematical aspects of two FI attacks and analyze the security of the implementation of the RSA in a digital signature scenario, in particular when the RSA-CRT algorithm is considered. By using the Chipwhisperer platform, we simulate the Bellcore [2] and the Lenstra [3] attacks to factorize the RSA module and to obtain the private key. The first attack compares a fake signature and the real one, whereas the second one only uses a wrong signature.

***K*eywords** RSA-CRT digital signature · Fault injection attack · Chipwhisperer · Bellcore and Lenstra attacks

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FINITE-TIMELESS LOCAL OBSERVABILITY FOR LINEAR CONTROL SYSTEMS ON LIE GROUPS

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ABSTRACT

Observability problem is one of the classical fundamental problem in the area and it is important from the applications point of view. The aim of this paper is to introduce a new approach to the observability problem in control theory. In this study, our approach comes from the finite-time point of view. For a general control system $\Sigma = (G, \mathcal{D}, h, V)$ on Lie groups, where G and V are Lie groups, $\mathcal{D}$ is the dynamic of the system and h is a smooth function between G and V, observability problem is to distinguish the points of the state space and the solutions of the system just by looking their image under h in V with the positive time.

In [1, 2], the authors study observability problem of linear control systems on Lie groups. In [1], the authors characterize local and global observability with a Lie group homomorphism and in [2], the authors characterize observability with a kind of projection. In [3], the author examines the existence of observable linear control systems on Lie groups appears in [1] and [2]. Recently, in [4], the authors give a comparison the observability properties of the class of linear control systems in two different manifolds: on the Euclidean space $\mathbb{R}^n$ in a more general setup, on a connected Lie group G. Related to these articles, a new time apporoach for the usual local observability is given.

In this study, linear control systems on connected Lie groups has taken into consideration and finite-timeless observability is introduced. For this new concept of observability, almost indistinguishability is defined and it has been shown that this is an equivalence relation. Also, properties of the image of almost indistinguishable points from the neutral element has studied. Finally, local observability is associated with finite-timeless local observability.

***K*eywords** observability · indistinguishability · linear control systems · Lie groups · Lie algebras

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THE P-FROBENIUS NUMBER FOR THE TRIPLE OF CERTAIN QUADRATIC NUMBERS

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ABSTRACT

We give closed-form expressions of the p-Frobenius number for the triple of the numbers $an(n-1)+r$ for an integer a and $|r|$ is odd. For the set of given positive integers $A := \{a_1, a_2, \ldots, a_k\}$, the p-Frobenius number is the largest integer whose nonnegative integral linear combinations of given positive integers in A are expressed in at most p ways. When p=0, the 0-Frobenius number is the classical Frobenius number, which is the central topic of the famous linear Diophantine problem of Frobenius.

***K*eywords** Frobenius problem · Frobenius numbers · quadratic numbers

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A REGULARITY CONDITION IN CONTINUOUS-TIME OPTIMIZATION PROBLEMS WITH INEQUALITY PHASE CONSTRAINTS

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ABSTRACT

Continuous-time optimization problems were introduced by Bellman [1] in 1953. Due to their importance in modelling various decision-making processes in the fields of robotics, mechanical engineering, aircraft control system design, and machine learning, continuous-time optimization problems have attracted considerable attention in the last 30 years, resulting in an extensive literature devoted to their various theoretical and computational aspects. The theory of scalar and vector continuous-time optimization has been intensively studied, and a large number of optimality conditions and dual models have been established. It should be noted that the validity of some results in this area has been questioned in recent research.

In this paper, we consider the nonsmooth and smooth case of a continuous-time optimization problem with inequality phase constraints defined in $L_\infty([0,T];\mathbb{R}^n)$. We establish new optimality conditions under an additional regularity condition and give some guidelines for obtaining optimality criteria of the Lyapunov-type extremal problem.

***K*eywords** Continuous-time programming · Optimality conditions · Convexity · Optimal control

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SEQUENCES OF BIPARTITE GRAPHS AND CONVERGENCE OF THEIR EIGENVALUES

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ABSTRACT

An interval in which a given graph has no eigenvalues is called a gap interval. We show that for any positive real number there exist infinitely many bipartite graphs with gap interval of a given length. We provide a recurrence relation for the computation of the characteristic polynomial of certain graphs and based on it, we conclude that the sequence of the least positive (resp. largest negative) eigenvalues of a growing sequence of these graphs is convergent. We also discuss possible limit points and their distribution on real line.

Keywords bipartite graphs · tridiagonal matrices · convergence

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THE SCHWARZ PROBLEM AND HIGHER-ORDER EXTENSIONS FOR THE BELTRAMI OPERATOR IN THE HALF UNIT DISC

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ABSTRACT

This study explores the Schwarz problem for the Beltrami operator within the upper half of the unit disc in the complex plane, a domain that is crucial for understanding complex analytic functions and their applications. We begin by deriving the solution to this boundary value problem, providing explicit expressions and necessary conditions for its solvability. This involves applying advanced techniques in complex analysis to solve the Schwarz problem and extending these results to second-order equations.

Additionally, we extend our findings to higher-order cases, which involves addressing more complex boundary value problems related to the Beltrami operator. This extension is achieved by generalizing the solution techniques used for the Schwarz problem and incorporating them into the analysis of higher-order differential equations.

***K*eywords** Beltrami Operator, Half Unit Disc, Schwarz Problem, Boundary Value Problems

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BOUNDARY VALUE PROBLEMS FOR THE BITSADZE EQUATION ON A QUARTER PLANE

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ABSTRACT

This study investigates boundary value problems for the Bitsadze equation defined on a quarter plane. The Bitsadze equation, a fundamental partial differential equation in the theory of analytic functions, often appears in complex analysis and mathematical physics. We consider specific boundary conditions that are either Dirichlet or Schwarz types on the respective edges of the quarter plane. By employing the method of integral transforms, we reduce the problem to a system of integral equations. The solvability of these equations is analyzed using analysis techniques. We further explore the uniqueness and existence of the solutions under different boundary conditions.

***K*eywords** Bitsadze equation, quarter plane, Dirichlet problem, Schwarz problem

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ON A TWO-STRAIN EPIDEMIC MATHEMATICAL MODEL WITH VACCINATION

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ABSTRACT

In this paper, we study mathematically a two strains epidemic model taking into account non-monotonic incidence rates and vaccination strategy. The model contains seven ordinary differential equations that illustrate the interaction between the susceptible, the vaccinated, the exposed, the infected and the removed individuals. The model has four equilibrium points, namely, disease free equilibrium, endemic equilibrium with respect to the first strain, endemic equilibrium with respect to the second strain and the endemic equilibrium with respect to both strains. The global stability of the equilibria has been demonstrated using some suitable Lyapunov functions. The basic reproduction number is found depending on the first strain reproduction number R_0^1 and the second reproduction number R_0^2. We have shown that the disease dies out when the basic reproduction number is less than unity. It was remarked that the global stability of the endemic equilibria depends, on the strain basic reproduction number and on the strain inhibitory effect reproduction number. We have also observed that the strain with high basic reproduction number will dominate the other strain. Finally, the numerical simulations are presented in the last part of this work to support our theoretical results.

***K*eywords** SEIR · COVID-19 · vaccination · non-monotone incidence

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GENERALISED TRIBONACCI HYBRID QUATERNIONS

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ABSTRACT

In the literature, many studies have been conducted on number sequences from the past to the present. Recently, the applications of these number sequences in different areas have accelerated. However, it has also been observed that there is an increase in the diversity of these number sequences. New sequences such as Leonardo, Narayana, Mersenne, Tribonacci and Nickel numbers have been added to these number sequences, which started with Fibonacci [3, 1, 7, 6, 2]. In this study, we will define generalized tribonacci hybrid quaternions using the Tribonacci sequence.

The generalised tribonacci sequence was first introduced in 1972 by Shannon and Horadam[8]. This sequence is the third-order recurrence relation, which is the most generalised form of numbers such as Tribonacci, Padovan-Perrin, Narayana, and 3rd-order Jacobsthal. The generalised tribonacci sequence is given as $\{V_n(V_0, V_1, V_2; r, s, t)\}$, where $r, s, t \in \mathbf{R}$ and $n \geq 0$. The n^{th} generalised tribonacci number is defined by the relation $V_n = rV_{n-1} + sV_{n-2} + tV_{n-3}$, [3]. The most well-known of these sequences are as follows:

- Tribonacci numbers $\{T_n(0, 0, 1; 1, 1, 1)\}$,
- Padovan-Perrin numbers $\{P_n(0, 1, 0; 0, 1, 1)\}$,
- 3. order Jacobsthal numbers $\{J_n(0, 1, 1; 1, 1, 2)\}$.

The nth generalised hybrid tribonacci number HV_n is defined by the relation

$$HV_n = V_n + V_{n+1}\mathbf{i} + V_{n+2}\varepsilon + V_{n+3}\mathbf{h},$$

where V_n is the nth generalised tribonacci number, and $\mathbf{i}, \varepsilon, \mathbf{h}$ are hybrid units [9]. Let HV_n be the generalised hybrid tribonacci number. Then after some necessary calculations, one can obtain the following recurrence relation:

$$HV_n = rHV_{n-1} + sHV_{n-2} + tHV_{n-3}, \quad n \geq 3$$

with initial conditions:

$$\begin{aligned}
HV_0 &= a + b\mathbf{i} + c\varepsilon + (rc + sb + ta)\mathbf{h}, \\
HV_1 &= b + c\mathbf{i} + (rc + sb + ta)\varepsilon + \left(\left(r^2 + s\right)c + (rs + t)b + rsa\right)\mathbf{h}, \\
HV_2 &= c + (rc + sb + ta)\mathbf{i} + \left(\left(r^2 + s\right)c + (rs + t)b + rsa\right)\varepsilon \\
&\quad + \left(\left(r^3 + 2rs + t\right)c + \left(r^2s + s^2 + rs\right)b + \left(r^2s + st\right)a\right)\mathbf{h}.
\end{aligned}$$

The generalised tribonacci quaternions is defined as

$$Q_{v,n} = V_n + V_{n+1}i + V_{n+2}j + V_{n+3}k, \quad n \geq 0.$$

where V_n is the n-th generalised tribonacci number [3]. Hybrid quaternions were introduced by Dağdeviren in [4] as a new number system. A hybrid quaternion is

$$HQ = Q_0 + \mathbf{i}Q_1 + \varepsilon Q_2 + \mathbf{h}Q_3$$

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such that Q_0, Q_1, Q_2 and Q_3 are quaternions. The other presentation of a hybrid quaternion is

$$HQ = H_0 + iH_1 + jH_2 + kH_3$$

such that H_0, H_1, H_2 and H_3 are hybrid numbers. Sum, subtraction and multiplication operations can be easily done with the information up to here. In the literature there are many other works about hybrid quaternions, we can refer ([5]). In this study, we first introduce a novel sequence associated with hybrid quaternions, referred to as the generalized Tribonacci hybrid quaternion, which has not been previously examined. Subsequently, we investigate the fundamental definitions and key properties of this newly defined sequence. Additionally, we present the addition operation, derive the Binet formula, and discuss various specific variants of the generalized Tribonacci quaternions. Finally, we describe some characteristics of particular types of generalized Tribonacci quaternions.

***K*eywords** Tribonacci numbers · Hybrid quaternion · Number sequences

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FIBONACCI AND LUCAS HYBRID SPLIT QUATERNIONS

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ABSTRACT

In this research, we begin by providing a concise overview of split quaternions and their generalizations involving complex, dual, and hyperbolic numbers. Subsequently, we introduce a new generalization called hybrid split quaternions and present their characteristics, including conjugate and norm. Furthermore, we explore their applications to Fibonacci and Lucas number sequences, along with providing generating functions and Binet's formula specific to Fibonacci hybrid split and Lucas hybrid split quaternions.

To understand the gap that this study aims to fill in the literature, we will summarize the existing research on split quaternions. Following this, we will present a table to thoroughly examine the primary objective of this study.

Split quaternions extend the algebra of standard quaternions by incorporating both real and hyperbolic components. While classical quaternions consist of four elements, including one scalar and three vector components, split quaternions are characterized by two real and two hyperbolic components. For real numbers z_i, a split quaternion can be expressed as $Q = z_0 + z_1 i + z_2 j + z_3 k$. In this formulation, the hyperbolic units j and k have the property that their squares equal 1. Split quaternions are particularly useful in physics and engineering, with applications in contexts such as Lorentz transformations and special relativity. The set of split quaternions is formally defined as:

$$\mathbb{H}_s = \left\{ Q = z_0 + z_1 i + z_2 j + z_3 k : i^2 = -1, j^2 = k^2 = 1, ijk = 1,\ z_i \in \mathbb{R} \right\}$$

The units i, j, and k exhibit non-commutative behavior, similar to that of the real quaternion units. However, split quaternions introduce unique algebraic characteristics, including the presence of nilpotent elements, zero divisors, and non-trivial idempotents [1, 2, 14].

Extensions of split quaternions are obtained by changing the component of the units. Lets have a split quaternion Q as $Q = z_0 + z_1 i + z_2 j + z_3 k$. If coefficients z_i are in complex number, dual number, and hyperbolic number then Q is called as complex split quaternions ($\mathbb{H}_s^{\mathbb{C}}$), dual split quaternions ($\mathbb{H}_s^{\mathbb{D}}$), hyperbolic split quaternions ($\mathbb{H}_s^{\mathbb{H}}$), respectively. This can be summarized with the following table.

Split Quaternions	Representation	Coefficients	References
$\mathbb{H}_s^{\mathbb{C}}$	$Q = Q_a + \mathbf{i}Q_b$	$z_i \in \mathbb{C}$	[8]
$\mathbb{H}_s^{\mathbb{D}}$	$Q = Q_a + \boldsymbol{\varepsilon}Q_b$	$z_i \in \mathbb{D}$	[3, 4, 9, 12, 16]
$\mathbb{H}_s^{\mathbb{H}}$	$Q = Q_a + \mathbf{h}Q_b$	$z_i \in \mathbb{H}$	[6, 7, 15, 17]
		$z_i \in \mathbb{K}$	-

Table 1: Extensions of Split Quaternions

In the later sections of the study, fundamental information on the original components will be presented, and split hybrid quaternions will be defined. This will enable us to examine the Fibonacci and Lucas number sequences within the established framework.

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Other studies related to the topic examined in our research, which we recommend to the reader, include [5, 13, 10, 11].

***K*eywords** Hybrid quaternions · Split quaternions · Fibonacci sequences

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BETWEEN τ^*-CLOSED AND $*IG$-CLOSED SETS

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ABSTRACT

One of the important basic notions in topological spaces is closed sets as well as open sets. Levine (resp. Jankovic et al. in 1990) introduced the notion of g-closed (resp. τ^*-closed) sets as two generalizations of closed sets. In this study, firstly we define a new closed set type called $*pIg$-closed is weaker than τ^*-closed and stronger than $*Ig$-closed. Then we obtain some properties of it. Similarly, we introduce a new type open sets called $*pIg$-open as complement of $*pIg$-closed and give some properties. Finally, we state when $*pIg$-closed is conserved.

***K*eywords** g-Closed Sets, ideal, τ^*-Closed Sets, $*Ig$-Closed Sets, $*pIg$-Closed Sets

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STUDY OF ELECTROMAGNETIC WAVE SCATTERING FROM AN IMPEDANCE COATED STEP IN A COAXIAL WAVEGUIDE

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ABSTRACT

Propagation and scattering problems of electromagnetic waves are encountered in various fields such as communication and radar systems, antenna applications, microwave filters. Therefore, there is a growing need to examine the scattering of electromagnetic waves from such structures with analytical methods. In this work, mathematical analysis of the propagation of the TEM wave from a coaxial waveguide with an impedance coated step discontinuity on its inner conductor is carried out by applying the Mode-Matching technique. In this method, a linear algebraic equation system consisting of infinite equations with infinite unknowns is obtained and this system is truncated at an appropriate truncation number. Finally, the reflected and transmitted fields are achieved.

***K*eywords** Boundary-Value Problems · Wave Propagation · Mode-Matching Technique

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This study was supported by Scientific and Technological Research Council of Turkey (TUBITAK) under the Grant Number 123E599. The authors thank to TUBITAK for their supports.

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STOCHASTIC NEIGHBOR EMBEDDING ALGORITHMS FOR DIMENSIONALITY REDUCTION

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ABSTRACT

Dimensionality reduction techniques are essential for the effective visualization and analysis of high-dimensional data. This study focuses on Stochastic Neighbor Embedding (SNE) and its variant, t-distributed Stochastic Neighbor Embedding (t-SNE) algorithms. These methods transform distances between observations into probability distributions using specific functions, while gradient descent is employed to minimize the Kullback-Leibler (KL) divergence between high-dimensional and low-dimensional representations. The principles behind these algorithms are explained, and their advantages and limitations are discussed.

***K*eywords** Stochastic Neighbor Embedding (SNE) · t-Distributed Stochastic Neighbor Embedding (t-SNE) · Dimensionality Reduction · Gradient Descent · Kullback-Leibler Divergence

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NUMERICAL INVESTIGATION OF BUILDING WALL STRUCTURE FOR A MEDITERRANEAN CITY

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ABSTRACT

Buildings are recognized as among the most energy-consuming systems, accounting for about 30% of the world's energy consumption and sources of greenhouse gas emissions. Improving building energy efficiency while maintaining occupant comfort has become increasingly important worldwide, as reflected in the growing stringency of building regulations. One significant passive energy control strategy is using insulation materials with low thermal conductivity. In this work, the thermal analysis of building wall structure is analyzed numerically by using the explicit finite difference technique. This conference paper examined the thermal performance of building multilayer walls composed of four layers: inner plaster, Hollow brick, insulation material (silica aerogel), and outer plaster, which concurrently provides better insulation. The outer layer of the building wall is subjected to combined convection and radiation. The inner building wall is exposed to the convection heat transfer. This analysis was carried out using actual weather data for 4 days in a city with a Mediterranean climate during summertime in Türkiye. The energy balance equations are written for each layer of the multilayer wall and discretized for each nodal point using the explicit finite difference method. The interface conductivity is calculated using the harmonic mean technique. The examined model is coded in the computer programming language MATLAB. This current research work determines the time-wise temperature variation and thermal performance of the building. The developed 1D numerical heat transfer model was validated with the published data. Moreover, the effect of the insulation material thickness on the thermal performance of the building wall is investigated. The results showed that the silica aerogel material has a substantial role in order to diminish energy consumption in the buildings.

Keywords Building wall, Explicit finite difference, Insulation material

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CODIMENSION-2 BIFURCATION ANALYSIS OF A MODIFIED NON-DEGENERATE FISHER EQUATION INTRODUCING TWO UNFOLDING PARAMETERS

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ABSTRACT

In this paper, we introduced a new modified Fisher equation with two-unfolding parameters formulated from the viewpoint of the field of singularity theory. We discussed the model Drichlet boundary to Neumann boundary. We checked the smoothness of the considered model and rigorously established the unfolding parameters based on codimension using the restricted tangent space. We find the solution by Lyapunov Schmidt reduction and we have discussed the bifurcation analysis of our proposed modified Fisher equation.

***K*eywords** Versal Unfolding · Restricted Tangent Space · Modified Fisher Equation · Non-degenaracy

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MATHEMATICAL MODELING OF BACTERIAL RESPONSE TO ANTIBIOTIC STRESS

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ABSTRACT

In this session, we try to understand how bacteria respond to antibiotic stress through mathematical analysis and modeling. Initially, we introduce a comprehensive set of ordinary differential equations that accurately illustrate the dynamics of bacterial growth and decay in response to antibiotic exposure. Subsequently, we explore another differential equations system designed to capture the interactions between antibiotics and nutritional factors. Finally, in the conclusion, we go over and discuss some computer simulations.

Keywords Antibiotic stress · Mathematical modeling · Computer simulations

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REGRESSION-BASED PREDICTION OF COMFORT IN SOFA DESIGN: A STUDY ON KEY DESIGN VARIABLES

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ABSTRACT

A sofa design is possible to be highly personalized depending on the users or their specific need for comfort. Besides the size, the key dimensions that significantly influence the perception of a sofa's comfort by the user include properties like seat depth, seat height, armrest height, overall width, and backrest height. It is very common for designers to approach these parameters with a view toward providing sofas in various contexts to cater to a wide array of preferences and body types. This study is a research into how such design factors can affect perceived comfort by testing 21 different models of sofas made by a market-leading furniture company. A questionnaire was applied to 69 participants for them to grade each of the sofa models with respect to comfort perceived. The participants rated their comfort experiences in relation to several attributes, such as seat height, arm breadth, seat depth, seat breadth, gap between arms, and back support offset. This was meant to give an overview of how multiple design elements impacted an individual's overall experience. The information from the participants was summarized into the development of a regression model that would predict comfort levels with the use of exact dimensions for a sofa. The input data have been filtered into the regression model on a scale ranging from 0 to 100 to allow normalization and maintain consistency. It is a standardization, which allows for avoiding potential bias due to the changing scale factor of various design parameters. The statistical software gave, as a result, a model with an impressive R-square value of 0.928, showing extremely strong relationships between sofa dimensions and comfort ratings given by participants. These findings can provide designers and manufacturers with better insights by showing quantitatively what improvements are needed to make sofas more comfortable. This study will help designers make informed decisions at the product development stage itself to improve user experience with the sofa. This adds to the existing literature on furniture design from an ergonomical point of view and underscores the importance of active participation by users in optimizing comfort.

***K*eywords** Sofa Design · Comfort · Statistical Analysis · Regression Model

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STRUCTURALLY FINITE CLASSES OF ORDER-PRESERVING FUNCTIONS IN THREE-VALUED LOGIC

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ABSTRACT

In this paper, we study structurally finite classes of order-preserving functions in the framework of three-valued logic (3VL). Specifically, we analyze the class $F \subseteq P_3$, where P_3 is the set of all functions in 3VL. A function class F is defined as structurally finite if it can be generated from a finite set of base functions. We focus on unary and binary order-preserving functions and introduce the notation $F(n)$ to represent the subset of functions in F depending on n variables.

Our main result shows that the classes $M(1)_3$, $KM(2)$, $DM(2)$, and $M(2)$ form a hierarchical structure in the lattice of order-preserving maps. We prove that the structural finiteness of these classes ensures that every function in $M \subseteq P_3$ can be generated by functions that depend on at most two variables:

$$KM(2) = K \cap M(2), \quad DM(2) = D \cap M(2).$$

Additionally, the overall lattice structure of these function classes can be depicted as:

$$M \supseteq M^{(2)} \supseteq \{D, K\} \supseteq \{DM^{(2)}, KM^{(2)}\} \supseteq M_3^{(1)}.$$

The structural finiteness of these function classes has important applications, particularly in optimizing systems like cellular networks (LTE, 5G) and compiler optimization, where a finite basis of functions simplifies algorithmic processes. These findings generalize results from Boolean logic to 3VL, opening avenues for further research in multi-valued logic systems and their applications in areas such as network traffic control, resource allocation, and computational logic [1, 2, 3, 4, 5, 6].

***K*eywords** Three-valued logic · Order-preserving functions · Structural finiteness · Multi-valued logic · Cellular network optimization

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FINITE GENERATED CLASSES IN k-VALUED LOGIC WITHOUT MAJORITY AND CHOICE FUNCTIONS

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ABSTRACT

This paper investigates the class of functions in k-valued logic, focusing on finite classes that do not contain majority and choice functions. Specifically, we analyze the class $F \subseteq P_k$, where P_k represents the set of all k-valued logic functions. A function class F is termed *structurally finite* if it can be generated from a finite set of base functions. We demonstrate that the class F, satisfying condition $0x$, is finitely generated and does not include majority or choice functions.

Our primary result establishes that the structural finiteness of the class F implies that all functions within it depend on at most $k-1$ variables. Formally, we show that:

$$F \subseteq T_0 \quad \text{where} \quad T_0 = \{f \in P_k \mid f(x_1, \ldots, x_n) = 0 \text{ if } x_i = 0\}.$$

Additionally, the absence of majority and choice functions in F is proven using the following result:

$$\mu \notin F, \quad \text{where} \quad \mu(x_1, x_2, \ldots, x_n) = \begin{cases} 1, & \text{if the majority of } x_i = 1, \\ 0, & \text{otherwise,} \end{cases}$$

and similarly for choice functions.

These results have significant implications in areas such as network traffic optimization and cellular systems (LTE, 5G), where reducing computational complexity and improving algorithmic efficiency are crucial. The established framework generalizes classical Boolean logic results to k-valued logic, with potential applications in the optimization of multi-valued logic systems [1, 2, 3, 4, 5].

***K*eywords** k-valued logic · Majority functions · Choice functions · Structural finiteness · Network optimization

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DERIVATIVE METHOD FOR SOLVING CUBIC EQUATIONS

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ABSTRACT

Cubic equations are important because of their application in different fields of engineering and science. There are different methods for solving cubic equations which have been completed through decades. The Babylonians, around 2000 BC, used geometric methods to solve cubic equations [1]. The Greeks attempted to solve cubic equations using the intersections of conic sections [2]. Al-Khwarizmi's work primarily focused on quadratic equations, but it laid the foundation for later Islamic mathematicians who tackled cubic equations [3]. Khayyam provided geometric solutions to cubic equations by intersecting conic sections. He classified cubic equations systematically [4]. del Ferro solved the depressed cubic equation but kept his solution secret until his death, passing it only to his student Antonio Fior [5]. Tartaglia discovered a method to solve the depressed cubic equation independently. His public challenge against Fior brought him fame [6]. Cardano published the solutions to cubic equations in "Ars Magna" (1545), which included Tartaglia's method and his contributions [7]. Ferrari, a student of Cardano, solved the quartic equation, which involves reducing it to a cubic equation [8]. Descartes' work in analytic geometry introduced methods to find the roots of polynomials, including cubic equations, using coordinates [9]. Abel and Galois developed group theory, which provided insights into the solvability of polynomial equations, including cubic equations, by radicals. Galois' work laid the foundation for modern algebra and provided criteria for determining the solvability of general polynomial equations [10]. All of these methods have some problems; some of them are too long and hard to memorize, some of them are not accurate and have errors, and some are not general. This article attempts to represent a method which has been easy to use and memorize and also accurate. In this article, we try to find a closed-form formula for deriving one of the roots of any cubic equation by using some substitutions. In other words, we use some substitutions to convert the cubic equation to a quadratic equation and then solve it with a quadratic formula (we call this method the Derivative method of solving equations, or the D-Method). This article just studies cubic equations, but the method used for deriving the formula may be generalizable to higher-degree (also lower-degree) equations, too.

***K*eywords** Cubic equation · Cubic formula · Derivative applications

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SOME RELATIONS INVOLVING ZEROS AND SPECIAL VALUES OF THE RIEMANN'S ZETA AND ALLIED FUNCTIONS

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ABSTRACT

Some recurrence relations for the Riemann zeta function $\zeta(s)$ at integer arguments, as well as relations involving nontrivial zeros of $\zeta(s)$ have been derived in [1]; for instance,

Proposition 1 ([1]) *For the Riemann zeta function $\zeta(s)$, it holds that*

$$\zeta(k)+\sum_{j=1}^{k-2}\lambda_j\zeta(k-j)+\gamma\lambda_{k-1}+k\lambda_k=0,\quad (k\geq 2),$$

where λ_k are the coefficients in the Taylor series expansion of $\tilde{\Gamma}(z)=1/\Gamma(1-z)$ around $z=0$, and γ is the Euler-Mascheroni constant.

Proposition 2 ([1]) *For the nontrivial zeros ρ of the Riemann zeta function $\zeta(s)$,*

$$\sum_{j=0}^{k-2}\lambda_j\left(\sum_{\rho}\frac{1}{\rho^{k-j}}\right)+\left[\frac{1}{2}\gamma+1-\log(2\sqrt{\pi})\right]\lambda_{k-1}+k\lambda_k=0,\quad (k\geq 2),$$

where λ_k are the coefficients in the Taylor series expansion of the Riemann ξ-function (or completed zeta function) around $s=0$, and γ is the Euler-Mascheroni constant.

The above propositions have been obtained using several formulas that were derived for the class of entire and meromorphic functions and that relate the sums of the nth powers of the reciprocals of zeros and poles of these functions with the coefficients of their Taylor series expansions [1], e.g.

Theorem 3 ([1]) *Let $f(z)$ be an entire function of finite order ρ, for which $p=\lfloor\rho\rfloor$, where p is the genus of f. Further, suppose that $\{a_n\}_{n\geq 1}$ is the sequence of zeros of $f(z)$. Then*

$$\sum_{j=0}^{k-p-1}\lambda_j\sigma_{k-j}+k\lambda_k=\sum_{j=k-p}^{k-1}\lambda_j(k-j)q_{k-j},\quad k>p$$

where λ_j are the coefficients in the Taylor series expansion of $f(z)$ at $z=0$, q_k are the coefficients of the polynomial in the Hadamard factorization of f, and σ_k are the sums of the form $\sigma_k=\sum_{n=1}^{\infty}1/a_n^k$. Moreover, the above assertion holds true when ρ is not an integer.

The aim of this talk is to discuss other recurrence formulas involving Riemann zeta function and some other allied functions [2]. In particular, similar results are established for the digamma function, Barnes G-function (or double gamma function), and its logarithmic derivative [3], which will be based on the results of the paper [1] and the Weierstrass infinite product representations of entire functions [3, 4]. A recurrence formula for the values of the Riemann zeta function at odd positive integers is also derived. I will also discuss the relations for the multiple gamma functions [5] and

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Vignéras' multiple gamma functions [6]. The determinantal formulas for the sums of the nth powers of the reciprocals of zeros of entire or meromoprhic functions as well as for the coefficients λ_k will also be presented. I will conclude by discussing some further research in this direction and outlining possible applications of the same ideas and the results obtained.

***K*eywords** Zeta functions · Sums of powers of reciprocals of zero · Entire functions · Weierstrass infinite product representations · Recurrence formulas

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DISCRETE-TIME REPLICATOR EQUATIONS AND EVOLUTIONARY DYNAMICS OF LEARNING IN OPTIMAL TRANSPORT NETWORKS

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ABSTRACT

The evolutionary dynamics of games, as systematically presented by Hofbauer and Sigmund [1], have broad applicability across numerous fields, including biology, ecology, economics, social systems, multi-agent frameworks, and learning theory. This interdisciplinary approach is rooted in applying dynamical systems theory to game theory, where various classes of dynamical systems – such as ordinary differential equations (e.g., the replicator equation), differential inclusions (e.g., best response dynamics), and reaction-diffusion models – serve as the mathematical backbone. A principal theme within this framework is the suboptimality of Nash equilibria in non-cooperative games concerning global system performance, quantitatively captured by the notion of the price of anarchy. This measure has received considerable interest in recent research.

In this talk, we first describe a novel application of replicator dynamics by formulating a discrete-time variant of continuous evolutionary dynamics. To this end, we introduce discrete-time replicator equations as a tool for analyzing optimal transport networks subject to congestion. Our presentation begins with the introduction of a Wardrop optimal network [2], which supports Wardrop equilibrium flows – flows that simultaneously satisfy Nash equilibrium conditions and achieve system-wide optimality; these networks are characterized by the price of anarchy equal to its least value, that is 1.

Building on this, we propose a dynamical model for optimal flow distribution in Wardrop optimal networks [2], leveraging the tools of evolutionary game theory, which places emphasis on the temporal evolution of strategies rather than static equilibria. Central to the analysis of evolutionary dynamics is the replicator equation, which quantifies the growth rate of agents adopting a specific strategy based on the deviation of the strategy's payoff from the population's average payoff. Strategies with above-average fitness increase in frequency, while those with below-average fitness decrease.

Our discrete-time dynamical model is based on mean-field replicator equations, defined on probability simplices, and generated by nonlinear, order-preserving mappings [3, 4]. In this talk, we focus on discrete-time replicator dynamical systems driven by Schur-convex potential functions [5], which will be demonstrated by using complete symmetric functions, gamma functions, and symmetric gauge functions as generators of the replicator dynamical systems. We study the system behavior, convergence, and stability properties, providing key insights into their temporal evolution. In particular, we examine the characteristics of Nash equilibria, the conditions for convergence to fixed points, and asymptotic stability within the replicator dynamics framework, utilizing tools from dynamical systems theory, such as Lyapunov functions. In the replicator systems under study, the Nash equilibrium, Wardrop equilibrium, and the system optimum converge to the same point in the state space, representing the flow distribution in the network.

We discuss the price of anarchy under different scenarios of transport network capacity and congestion parameters, along with the total costs at Nash equilibrium and system optimum. This analysis helps in understanding the dynamics of transportation networks and optimizing the learning rate for effective convergence to the Nash equilibrium. Our results demonstrate the significant impact of learning rates on the convergence behavior of replicator dynamics in transportation networks.

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To analyze the dynamics of learning using Poincare sections, we consider a multi-agent transportation system, where agents adapt their strategies over time based on the discrete-time replicator equations, both deterministic and stochastic. We present the bifurcation diagrams for evolutionary dynamics of learning in case of stochastic replicator equations. These diagrams provide valuable insights into the dynamic behavior of the stochastic replicator equations model. By analyzing these diagrams, one can understand how the system transitions between different regimes of stability and chaos as the learning parameter is varied, which is crucial for designing optimal transport networks, where learning dynamics play a significant role. We also present the simulation results that validate the theoretical analysis, including the examination of convergence rates for the orbits of the replicator system to fixed points under various function types generating the replicator dynamics.

Finally, we emphasize that potential applications of the proposed replicator dynamical model include applications to neural network models and the analysis of learning in neural network dynamics [6].

***K*eywords** Replicator equation · Evolutionary dynamics · Wardrop optimal network · Replicator dynamical system · Learning dynamics · Stability · Equilibrium · Optimum

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A COMPARISON OF ALGORITHMS FOR ENTROPY MEASURES IN ELECTROENCEPHALOGRAPHY

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ABSTRACT

Quantifying the amount of uncertainty or randomness, thus a level of new signal pattern generation in biomedical data, becomes an effective way of unraveling hidden mechanisms responsible for their alterations. Analysis of nonstationary signals with a non-linear nature requires appropriate analytical tools to characterize dynamical brain states. In this study, we compare approximate, sample, permutation, or wavelet entropy measures to monitor pathophysiological changes of EEG that correlate with clinical observations and have the potential to serve as an effective diagnostic and prognostic tool.

***K*eywords** Neuroscience · Approximate Entropy · Sample Entropy · Permutation Entropy · Wavelet Entropy

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PROBLEM-BASED LEARNING AND DEVELOPMENT OF CORE SKILLS IN NUMERICAL METHODS

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ABSTRACT

In mathematics courses, it is common practice to assign students problems that align with the mathematical concepts and competencies they are expected to acquire. This research explores the connection between two core skills, teamwork and entrepreneurship, within the context of problem-based learning. In the Numerical Methods course for Industrial Engineering students, problem-solving is a core component. While many students have prior experience working in teams, fewer have been exposed to developing entrepreneurial skills. Teamwork involves the ability to interact, collaborate, and cooperate effectively with others, recognizing the unique value of each individual. This core skill is rooted in self-awareness and understanding others through shared experiences. The entrepreneurial skill addresses contemporary challenges and is increasingly valued in higher education. Entrepreneurs are driven by the desire to address real-world problems and find creative solutions. Problem-based learning involves small groups of students working collaboratively to solve complex problems. This peer-to-peer learning approach fosters knowledge sharing and critical thinking.

This study analyzes the relationship among problem-solving in Numerical methods course and the development of two specific core skills such as teamwork and entrepreneurship.

***K*eywords** Problem-based learning · core skills · teamwork · entrepreneurship

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EVALUATING CUSTOMER CHURN ANALYSIS VIA DATA MINING

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ABSTRACT

Customer churn, which occurs when customers stop using a company's products or services, represents a significant challenge for businesses in sectors like banking, telecommunications, and insurance. Retaining customers is crucial for long-term profitability, as acquiring new customers often incurs higher costs than keeping existing ones [1]. To address this issue, companies are increasingly turning to data mining techniques to predict customer churn and devise proactive retention strategies [2].

Customer churn is influenced by various factors, including customer behavior, service usage, satisfaction levels, and external market conditions[3]. Predictive analytics offers a powerful approach to identifying at-risk customers before they leave. Commonly used methods for churn prediction include Logistic Regression, Classification models, Artificial Neural Networks, and a range of machine learning algorithms such as decision trees, support vector machines (SVM), and clustering techniques[4][5]. These methods help businesses process large datasets, identify patterns, and make informed decisions about which customers may churn[6].

In this study, we aim to enhance the understanding of customer churn in the banking sector by applying a combination of data mining techniques. While previous research has focused heavily on industries like telecommunications and retail[7], fewer studies have explored churn prediction specifically within banking[8]. This study fills that gap by employing a diverse set of methods, including Logistic Regression, Artificial Neural Networks, and machine learning models, to evaluate their effectiveness in predicting customer churn in banking[9].

Our approach distinguishes itself from existing research in two key aspects. First, we provide a detailed comparison of different data mining techniques to assess their performance in identifying at-risk customers in the banking industry[10]. Second, we focus on understanding the specific drivers of churn within this sector, offering banks practical insights to not only predict but also address the root causes of customer attrition. By applying and comparing multiple techniques, this research provides a comprehensive evaluation, empowering banks to improve their customer retention strategies through data-driven insights.

***K*eywords** Customer churn · Data mining · Machine learning · Banking

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ON EXPLORING THE EQUIVALENCE OF THREE DIFFERENT CLASSES CONSISTING PRIME PAIRS: THE TWIN PRIME PATTERNS SURROUNDING LARGE EVEN INTEGERS

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ABSTRACT

The 'Goldbach conjecture' simply states that, every even integer (≥ 6) can be expressed as the sum of two primes. Mathematicians have tried to prove it analytically, from the centuries. Unfortunately, all such attempts, leaving us with void certainty. We have utilized the 'Goldbach's conjecture' to extend it. In this study, we've strengthened the conjecture by imparting two hypotheses. Our objective relies upon the necessity of 'unveiling the twin prime pattern with even integers'. In this article, we have illustrated two hypotheses, based on our intriguing observations. We insist that, in the sum of two primes that represent an even integer (≥ 6), at least one of these primes must belong to a twin prime pair. For convenience consider three numerical examples: Taking $8 = (3 + 5)$. Here (3,5) constitutes twin prime pair. For $20 = (3 + 17)$, where (3,5) and (17,19) constitutes twin prime pairs independently. For taking $52 = (5 + 47)$, where 5 is part of a twin prime pair (5,7), but 47 is isolated prime. On further exploration, we have shown, the sum of twin prime pair representing an even integer ($\geq$36); can be expressible as sum of primes (both primes individually constitute twin prime pair), and sum of primes (one isolated prime, another prime constitute twin prime pair). For instance, consider the twin prime pair (617,619), on addition $(617+619) = 1236 = (5+1231) = (1193+43)$; (5,7), (1229,1231) are individual twin prime pairs; 1193 is an isolated prime, (41,43) is a twin prime pair. To establish the truthfulness, we've conducted extensive computational checks on these hypotheses. Examining numbers up to , our results confirm its validity (using Python software). Yet, like its predecessor, the Goldbach Conjecture, proving these hypotheses mathematically is likely to be an exceptionally challenging endeavor.

***K*eywords** Primes · The Goldbach's Conjecture · The Twin Primes

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SOME LINEAR ALGEBRAIC PROPERTIES OF TOEPLITZ-TYPE MATRICES

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ABSTRACT

In this talk, we study the basic linear algebraic properties of some matrices of Toeplitz-type whose entries are generalized Fibonacci and Lucas numbers using some matrix theory methods. We also compare our results with some results in the literature.

***K*eywords** Determinant · Inverse · Toeplitz matrix

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AN EMPIRICAL COMPARISON OF SUPERVISED MACHINE LEARNING MODELS IN PREDICTING MATHEMATICS PERFORMANCE: A CASE STUDY OF THE 2022/2023 SOMALILAND NATIONAL PRIMARY EXAMINATION RESULTS

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ABSTRACT

This study examined the factors affecting mathematics performance among primary school students in Somaliland using the 2022/2023 Somaliland National Examination results. The analysis revealed a significant decline in mathematical performance, with the failure rate increasing from 51.9% in 2020 to 42.3% in 2023. Regions and districts showed significant variations in mathematics performance, with regions such as Awdal and Maroodi Jeeh having the highest failure rates. District-level variations were also observed, with Sheekh achieving high pass rates of 98.95% compared with lower rates in Borama and Hargeisa. Gender disparities were also evident, with male students having a higher failure rate (67.17%) than that of female students (63.52%). Urban schools had a higher failure rate (67.64%) than did rural schools (45.21%). Public schools had a lower failure rate (62.21%) than private schools did (68.08%). Six machine learning models were used to analyze predictors of mathematics performance, with Naïve Bayes and KNN being the most accurate models, with an accuracy rate of 98.6% and 80.3%, respectively.

***K*eywords** mathematical performance · machine learning · logistic regression · prediction.

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