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A SINGLE-PROJECTION PROXIMAL ALGORITHM FOR STOCHASTIC MIXED VARIATIONAL INEQUALITIES WITH APPLICATIONS TO BREAST CANCER SCREENING

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ABSTRACT

We studied a stochastic mixed variational inequality problem (SMVIP) that encompasses stochastic optimization, stochastic variational inequality problems, and a composite convex minimization problem as special cases. To solve this problem, we proposed a single-projection proximal algorithm (SiPPA) that combined golden ratio dynamics with an adaptive stepsize strategy. In contrast to classical stochastic extragradient and subgradient extragradient methods, the proposed algorithm required only one projection and one averaged stochastic oracle call per iteration, resulting in reduced computational cost. Under mild assumptions on the stochastic oracle and monotonicity of the expected operator, we established almost sure convergence of the generated sequence. Moreover, when the operator was strongly monotone, we proved that the algorithm converges at an $R-$linear rate. Numerical experiments on benchmark problems and real-world learning tasks on breast cancer screening, illustrate the effectiveness and efficiency of the proposed approach relative to existing stochastic methods.

Keywords Projection. Golden ratio. Mixed variational inequality. Stochastic optimization. Almost sure convergence. $R-$linear convergence.

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