VII International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2026)

ICMASE 2026

MACKAY'S APERIODIC TILING

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ABSTRACT

In this talk we shall describe the historical context and the basic properties of a relatively unknown early aperiodic tiling with two prototiles. It was presented by the British crystallographer Alan Lindsay Mackay (1926–2025) in 1975 [4], when he described his ideas on the possibility of highly regular structures, but not periodic like "normal" crystals — at that time only a conjecture, but in 1982 proven to be possible by the discovery of quasicrystals [5]. Mackay's tiling was intended as a model of a two-dimensional aperiodic hierarchical structure based on regular pentagons. On a first glance his construction has an infinite set of prototiles of two types: congruent regular pentagons, and golden triangles unlimitedly increasing in size. However, Mackay provided a graphical hint of substitution rules (for regular pentagons and golden triangles), that suggests that the same pattern is obtainable using two prototiles: a golden triangle, and a golden gnomon. He never published the details of his construction, but his method indeed results in an aperiodic substitution tiling of the plane using only two prototiles [2, 3, 4, 5].

Here we shall give the historical context of Mackay's discovery, discuss the reasons why Mackay's tiling until now received little attention in mathematical literature on aperiodic tilings, and provide arguments that his discovery was independent from the other more famous aperiodic sets with two tiles (Penrose's, Robinson's, ...) that were discovered at about the same time [2, 3]. Furthermore, we complete his construction with a proof that his tiling is indeed always an aperiodic substitution tiling with two prototiles, we describe the necessary but originally missing substitution rules for golden gnomons, and show that all such tilings have ϕ^2 as inflation factor [1].

***K*eywords** aperiodic tilings · history of geometry · quasicrystals · Alan Lindsay Mackay · mathematical crystallography

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