
A NOVEL NUMERICAL APPROACH FOR FRACTIONAL FISHER–KPP AND ROSENAU–HYMAN DYNAMICS

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ABSTRACT

In this manuscript, we develop a novel analytical scheme as an extension of the q-homotopy analysis method (q-HAM), referred to as the optimal q-HAM, for solving fractional diffusion equations involving a recently introduced fractional derivative with a singular Prabhakar kernel. This fractional derivative is constructed as the inverse operator of a generalized fractional integral associated with the Prabhakar kernel, enabling a more accurate description of memory-dependent diffusion processes. To derive approximate analytical solutions, we formulate an optimal auxiliary linear operator using a Maclaurin polynomial-based approximation, which significantly enhances the convergence characteristics of the q-HAM. The proposed methodology systematically controls the convergence through an optimal convergence-control parameter, thereby improving numerical accuracy and computational performance. Three numerical problems, including the fractional Fisher-Kolmogorov-Petrovsky-Piskunov (Fisher-KPP) and Rosenau-Hyman equations, are investigated to evaluate the accuracy, convergence behavior, and computational efficiency of the proposed scheme. A comparative analysis with the q-HAM demonstrates that the optimal q-HAM substantially reduces computational complexity while achieving faster convergence and higher solution accuracy. Numerical simulations, accompanied by 2D and 3D graphical visualizations and comprehensive error analysis, validate the robustness and effectiveness of the proposed framework.

Keywords Fisher–KPP Equation · Rosenau-Hyman Equation · Caputo Derivative · Prabhakar-kernel Based Fractional Derivative · Optimal q-HAM

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