
A GENERALIZATION OF R-SUPPLEMENTED MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left R -module. Let M be an R -module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N + L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and V is minimal with respect to this property, or equivalently, $M = U + V$ and $U \cap V \ll V$, then V is called a supplement of U in M . M is called a supplemented module if every submodule of M has a supplement in M . The intersection of all maximal submodules of an R -module M is called the radical of M and denoted by $RadM$. If M have no maximal submodules, then the radical of M is defined by $RadM = M$. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll M$, then V is called a weak supplement of U in M . M is said to be weakly supplemented if every submodule of M has a weak supplement in M . Let M be an R -module and $K \leq M$. If $K \ll RadM$, then K is called an r -small submodule of M and denoted by $K \ll_r M$. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_r V$, then V is called an r -supplement of U in M . If every submodule of M has an r -supplement in M , then M is called an r -supplemented module. Let M be an R -module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_r M$, then V is called a weak r -supplement of U in M . If every submodule of M has a weak r -supplement in M , then M is called a weakly r -supplemented module. In this work, some properties of weakly r -supplemented modules are investigated. It is clear that every r -supplemented module is weakly r -supplemented. Because of this weakly r -supplemented modules are more general than r -supplemented modules.

Keywords Small Submodule · Radical · Supplemented Module · r -Supplemented Module

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