GEODESIC CURVATURE RELATIONS OF CURVE PAIRS SHARING NORMAL DIRECTIONS ON S^2

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ABSTRACT

Focusing on the viewpoint of geodesic curvature, this paper explores the intrinsic geometric links between two curves rolling without slipping on the unit sphere S^2. The phenomenon of rolling motion without slipping introduces rigid structural constraints, which we systematically investigate by examining specific curve pairs characterized by a shared geodesic normal direction at their points of contact. Rather than employing standard kinematic formulations, this study adopts a purely differential geometric approach. First, the explicit coordinate-free relations and transformations between the geodesic Frenet frames of these matching curves are established. Building upon these fundamental frame transformations, we subsequently analyze and derive the exact mathematical relationships and identities governing their respective geodesic curvatures. Furthermore, we evaluate the geometric implications of these curvature dependencies under various rolling conditions. The theoretical framework developed herein not only provides deeper insights into the intrinsic geometry of spherical curves but also offers a robust mathematical alternative for analyzing constrained rotational motions on non-Euclidean surfaces.

***K*eywords** Spherical curves, Geodesic curvature, Motion

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