

## THE DISTRIBUTION OF $\alpha p^{\gamma} + \beta$ MODULO ONE AND *r*-FREE NUMBERS

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## ABSTRACT

Let  $\alpha, \beta \in \mathbb{R}$  be given. Assume that  $a_1, a_2, \ldots, a_n$  are distinct positive integers and let  $\mathcal{P}$  be the infinite set of prime numbers for which  $a_1, a_2, \ldots, a_n$  does not form a reduced residue system modulo  $p^r, r > 2$  for any  $p \in \mathcal{P}$ . For any fixed  $\gamma$  with  $0 < \gamma < 1$ , we prove that there are infinitely many primes  $p \in \mathcal{P}$  such that  $||\alpha p^{\gamma} + \beta|| < p^{-\theta}$  and all the numbers  $p + a_1, \ldots, p + a_s$  are free of rth powers.

**Keywords** Distribution modulo one  $\cdot r$ -free numbers  $\cdot$  estimates of exponential sums

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