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A Solution to the Fourth-Order Nonlinear Problem with Symmetric Boundary Conditions Using the Banach Fixed Point Theorem and Green's Function

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Abstract

In this study, a fourth-order nonlinear differential boundary value problem is considered. The problem has a structure in which the solution function depends on both itself and its first derivative, and it is defined by symmetric mixed boundary conditions. First, the Green's function appropriate to the problem is constructed. Then, the existence, uniqueness, and iterative convergence of the solution are proven using the Picard iteration method in conjunction with the Banach fixed point theorem. Theoretical results are supported by a numerical example, and it is observed that the iteration process converges rapidly..

***K*eywords** Banach Contraction Principle · Green Function · Fouth Order BVP

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