

# ON THE $(r, p, k)$ -GENERALIZED JACOBSTHAL NUMBERS AND THE JACOBSTHAL FUNDAMENTAL FIBONACCI SYSTEM

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## ABSTRACT

Recently, several research papers has been produced on the classic sequences of additive theory such as the Fibonacci-Lucas numbers, the Pell numbers and the Jacobsthal numbers. Although they are theoretical in nature, these research papers were motivated by the numerous applications of these sequences of numbers and their various generalizations, in different mathematical and applied mathematics fields, as well as in the exact and applied sciences, and in art. (see, for example, [2, 3] and references therein). This talk deals with a model of  $(r, p, k)$ -generalized Jacobsthal numbers by the approach based on the properties of its associated Jacobsthal fundamental Fibonacci system. Some linear and combinatorial properties are established. Moreover, the related matrix formulation allows us to provide new identities. Especially, the linear Jacobsthal Cassini identity and its combinatorial formulation are furnished. Finally, illustrative special cases and examples are given. More precisely, the model of  $(r, p, k)$ -generalized Jacobsthal numbers is defined by

$$\begin{cases} J_{n+1} = J_n + 2^p J_{n-1} + J_{n-2} + \cdots + J_{n-r} + 2^k J_{n-r+1}, & \text{for } n \geq r, \\ J_0 = \alpha_0, \cdots, J_{r-1} = \alpha_{r-1}, \end{cases} \quad (1)$$

where  $\alpha_0, \cdots, \alpha_{r-1}$  are the initial conditions and  $k, p$  are given integers in  $\mathbb{N}$ . We can observe that for  $p = 1$  and  $k = 0$  or  $p = 0, k = 1$  in (1), we get known following generalized Jacobsthal numbers

$$J_{n+1} = J_n + 2J_{n-1} + J_{n-2} + \cdots + J_{n-r} + J_{n-r+1}, \quad \text{for } n \geq r, \quad (2)$$

$$J_{n+1} = J_n + J_{n-1} + J_{n-2} + \cdots + J_{n-r} + 2J_{n-r+1}, \quad \text{for } n \geq r, \quad (3)$$

where  $J_0 = \alpha_0, \cdots, J_{r-1} = \alpha_{r-1}$  are the arbitrary initial conditions. In addition, for  $p = k = 0$  Expression (1) is reduced to the well known generalized Fibonacci numbers of order  $r$  studied in various research papers (see, for example, [1, 4], and references therein).

**Keywords** Generalized Jacobsthal numbers ·  $(r, p, k)$ -Generalized Jacobsthal numbers · Jacobsthal Fundamental Fibonacci System · matrix representation · combinatorial formulas · Jacobsthal Cassini identity.

## References

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